Hidden Action, Identification, and Task Assignment

Wendelin Schnedler*
Department of Economics
University of Heidelberg

First Version: September 2008
This Version: September 2011

Abstract

Incentives often fail in inducing economic agents to engage in a desirable activity. Sometimes this restriction on implementability can be overcome by assigning tasks differently. This paper shows that any restriction of implementability is caused by an *identification problem*, describes task assignments that can solve this problem and provides conditions under which such assignments exist. Applying the findings to established and new moral hazard models yields insights into optimal task assignment, uncovers the reason why certain task assignments, such as advocacy or specialization, overcome restricted implementability, and formalizes a wide-spread type of multi-tasking problem.

keywords: moral hazard, hidden action, implementation, multi-tasking, identification by task assignment

JEL-Codes: D86, M52, J33, D82, M41

*wendelin.schnedler@awi.uni-heidelberg.de, Heidelberg University, Department of Economics, Bergheimer Straße 58, D-69115 Heidelberg, Germany
The author wishes to thank Stanley Baiman, Matthias Blonski, Adam Dominiak, Jürgen Eichberger, Guido Friebel, Paul Heidhues, Alexander Koch, Sebastian Köhne, Michael Kosfeld, Jean-Phillipe Lefort, Paul Milgrom, Dilip Mockerjee, Marc Möller, Julia Nafziger, Nick Netzer, Jörg Oechssler, Michael Raith, Andreas Roider, Richard Saouma, Patrick Schmitz, Heiner Schuhmacher, Uwe Sunde, Christoph Vanberg, Elu von Thadden, and Georg Weizsäcker for valuable discussions. The paper also benefited from comments of participants at seminars in Bristol, Mannheim, Frankfurt, and Paderborn as well as at the annual meeting of the European Economic Association 2009 in Barcelona, the Accounting Research Workshop 2009 in Bern, the Gaming Incentive Systems Conference 2011 in Bonn, and the annual conference of the European Association of Labor Economics 2010 in London.

Introduction

Often, it is hard to get economic agents to engage in desired activities. In legendary examples, incentives based on number of lines programmed led to longer but not necessarily better programs at AT&T and secretaries being paid by the number of key strokes were found hitting the space bar during lunch break at General Electric.[1] Renown economic models illustrate that better quality cannot be obtained from a builder when only the speed of construction is observable (Holmström and Milgrom, 1991) and that a single investigator slacks with respect to finding either exculpatory or incriminating evidence when incentives can only be attached to the court's ruling (Dewatripont and Tirole, 1999). In all these examples, it is impossible to provide incentives based on the available observable outcomes (signals) such that agents act in a desired manner; implementability is limited. However, by assigning tasks differently this restriction can sometimes be overcome. For example, Ciba Geigy succeeded in generating more innovative products only after dividing their research and development department[2] and Dewatripont and Tirole famously proposed to obtain exculpatory and incriminating evidence by having two advocates rather than a single investigator. In these examples, task assignment affects whether desired behavior can be induced and hence the loss from being unable to directly stipulate the desired activity

[1] More recently, Ben Arnoldy reported in the Christian Science Monitor (28th of July 2010) on attempts to reduce the negative effects of the Colorado potato beetle in Afghanistan by paying farmers 5$ for each bottle filled with such beetles; as a consequence, some farmers started breeding them.

[2] See "The Ambidextrous Organization" by O'Reilly and Thushman in Harvard Business Review April 2004.

(agency costs). Institutions, contracts, etc. that minimize agency costs can hence not be understood fully without studying implementability and how it depends on task assignment.

The literature offers no comprehensive view on why and when task assignment affects implementability. In the advocates model, the effect is usually attributed to the conflicting nature of the two desired types of effort on the signal (see, e.g, Dewatripont, Jewitt, and Tirole 2000 or Bolton and Dewatripont 2005). However, Ratto and Schnedler (2008) show that task separation can also induce desired behavior if tasks do not conflict. Corts (2007) argues that switching from individual to team accountability 'enriches signals.' As argued in Section 6.2, such a change in accountability is not essential either. The aforementioned legendary examples of limited implementability have been associated with the fact that agents engage in more than one task (multi-tasking)–see for example Prendergast's well-known survey on incentives (1999). However, as will become clear later, multi-tasking neither causes limited implementability in the legendary examples nor in Holmström and Milgrom's seminal article on multi-tasking (1991).[3]

In summary, while it is clear that task assignment can affect agency costs by changing which activity choices can be induced, it remains to be explained (i) what restricts implementability in the first place, (ii) how task assignment affects this restriction, (iii) how the restriction relates to multi-tasking, and (iv) when task assignments can overcome the restriction. The present article sheds some light on these issues.

[3]The respective argument in Section 6 clarifies, supports, and extends Cort's claim (2007) that (once randomness is removed) Holmström and Milgrom feature 'clearly no multitask problem.'

The crucial notion to understand the effect of task-assignment on implementability is that of *conditional identification*. Given a specific task partition, a generic activity choice is implementable if and only if each agent's behavior can be identified from signals given that all other agents do not change behavior (Theorem 1). This notion of identification is different from respective notions in econometrics or the literature on partnerships because of its conditional character:[4] it suffices that it would be theoretically possible to infer what some agent did *if one were to know the choices of other agents.*

Precisely because of the conditional nature of identification, it becomes possible to construct a game in which activity choices form a Nash equilibrium. For the intuition, consider deterministic signals. Holding constant the behavior of others, conditional identification ensures that every signal realization can only be produced in one way; by attaching rewards to the realizations associated with some desired behavior, this behavior can be induced. Task assignments affect whether activities are conditionally identified and hence agents' scope to generate signals. Here, this intuition is formalized for stochastic signals using marginal analysis.[5]

Apart from upsetting the trade-off between incentives and insurance (Holmström and Milgrom, 1991), multi-tasking increases agents' scope and may hence cause identification problems and restrict implementability. This type of multi-tasking problem is necessary (and sufficient) for task assignment to overcome identification problems (Corollary 3). This observation directly

[4]The detailed relationship to these notions is discussed after the formal definition of conditional identification in Section 3.

[5]Whether or not marginal analysis is employed is immaterial for the link between identification and implementability (Theorem 1): Proposition 5 offers a formalization without relying on marginal arguments.

implies a straightforward characterization: identification by task assignment is possible if and only if choices at some task l can be inferred from signals while holding constant the choices at all other tasks (Corollary 4). In short, signals have to be task-wise injective.

The link between identification and implementability established here is based on two propositions that are interesting in their own right. First, a specific activity can be implemented if and only if the activity's marginal effect on agents' costs are a linear combination of that on signals (Proposition 1). This result generalizes earlier findings by Feltham and Xie (1994) and Corts (2007). A direct corollary confirms that assuming contracts to be linear is without loss of generality when trying to determine whether an activity choice is implementable (Corollary 1), which contrasts with Mirrlees (1999) famous observation that assuming linearity is not innocuous when finding optimal contracts. Second, activities are conditionally identified given a partition of tasks if and only if each agent faces at most as many tasks as independent signals (Proposition 2), where independence is measured by the (minimum) rank of the matrix that describes the marginal effects of the respective agent on signals. Task assignment thus 'enriches' signals if it increases the number of independent signals relative to that of tasks such that activities become conditionally identified.

The next section relates our findings to the literature. Section 2 illustrates the core concepts and main findings with a simple example. Section 3 describes the hidden action framework to which results apply and which nests most moral hazard models as special cases; it also formally defines implementability and identification. Section 4 then establishes that any activity can be implemented if and only if it is (conditionally) identifiable.

Section 5 provides necessary and sufficient conditions for identification by task reassignment. Section 6 contains various examples of established and new moral-hazard models in which the results on implementability help to understand the nature of optimal contracts. Section 7 concludes.

1 Related Literature

The notion that identification is crucial for implementability is implicit in various single-task models. Hermalin and Katz's observe (1991) that a specific activity choice is only implementable if there is no (less costly) way for an agent to produce the same signal distribution—see their Proposition 2. Fudenberg, Levine, and Maskin (1994) find that in a repeated game setting with discrete and finite signal realization, agents can be induced to engage in desired activity choices if distributions vary with the choice (individual full rank condition). While there are several differences to these contributions,[6] probably the most important innovation here is the introduction of multiple tasks, which is necessary to study the effect of task assignment on implementability.

Implementability has been studied in multi-tasking models under the restriction that signals and contracts be linear (Feltham and Xie, 1994; Corts, 2007); the respective results are generalized here to non-linear signals (Proposition 1) and the approach of limiting attention to linear contracts is justified (Corollary 1). The paper is inspired by and closely related to Corts (2007),

[6]For example, implementation is reflected in Fudenberg, Levine, and Maskin (1994) by an indifference condition, whereas here it manifests in a first-order condition. The ramifications of this difference are explained later in the analysis.

who also examines the effect of task assignment—albeit without uncovering the central role of conditional identification. Here, the tacit mechanism underpinning his results as well as others (see e.g. Dewatripont and Tirole, 1999) is uncovered. The generality of the findings allows for a host of new applications; some of which are sketched later.

Following Holmström's description of the free-rider problem (1982), several contributions ask when it is possible to obtain efficient efforts in partnership problems (see e.g. Legros and Matsushima, 1991; Strausz, 1999; Battaglini, 2006). Partnership problems differ from the setting examined here in two important ways. First, in partnership problems, signals and output are identical, so that it is efficient to produce signals at the lowest costs for agents. The possibility that signals badly reflect outputs, which is a major concern in multi-tasking models, is excluded. Accordingly, there is no discrepancy between agents' preferred and the optimal way to generate signals. On the other hand, there is the difficulty that the budget must balance for all signal realizations. This difficulty can be overcome if realizations allow for identifying at least one agent who has not deviated (an 'innocent agent'); deviating agents can then be punished by not letting them participate in the revenue but handing it to the innocent agent. Using this idea, Battaglini (2006) challenges conventional wisdom by showing elegantly that team size does not matter for inducing an efficient allocation in partnership problems. Rather, it is the average number of tasks per agent in relation to the number of signals. Moreover, task assignment plays no role, which sharply contrasts with its importance here. The differing research question asked in this literature thus leads to different results.

Some of the issues in the present paper, e.g., implementability or task assignment, are also relevant for mechanism design with hidden information.[7] They are examined here from a hidden action perspective. Finally, organization design in general and task assignment in particular can also be discussed outside the principal-agent paradigm—see Borland and Eichberger (1998) for an overview.

2 A Simple Example

The following example is strapped off all bells and whistles to illustrate the results and underlying concepts and intuitions.[8] In order to successfully produce and sell a computer program, three tasks are required: programming, debugging, and marketing. Each of the tasks involves an unobservable (real-valued) effort choice: $a = (a_1, a_2, a_3)$. Programming and debugging affect the stability of the program, $\mu_1 = a_1 + a_2$ about which there is a noisy contractible signal $S_1 = \mu_1(a) + \epsilon_1$, where ϵ_1 is some continuously distributed random variable (linearity is not essential and only assumed for expository reasons). Program sales can also be observed and depend on the program's stability, μ_1, marketing effort, a_3 and a random variable, $\epsilon_2 : S_2 = \mu_2 + \epsilon_2$, where $\mu_2 = \mu_1 + a_3$.

Now consider different ways to split the three tasks $\{1, 2, 3\}$. Let there be

[7] A classical problem is that of implementing a social choice rule (see e.g. Fudenberg and Tirole, 1991, Chapter 7.1). Mookherjee (2006) provides an excellent survey on organizational structures in hidden information models, in particular on the question when decisions should be decentralized. Alonso and Matouschek (2008) are concerned with limiting choice sets if agents are better informed; Raith (2008) examines how to provide incentives to work to such agents.

[8] The example is inspired by Ratto and Schnedler (2008).

a production department, $l = P$, that decides on programming effort and a marketing department, $l = M$, that decides on marketing effort. The debugging task may be located in either of the two departments. If the production department decides on programming and debugging, (a_1, a_2), while the marketing department only decides on marketing, a_3, this is represented by the partition: $\{\{1,2\},\{3\}\}$. Conversely, if the production department decides on a_1, while the marketing department debugs and markets, (a_2, a_3), the task partition is $\{\{1\},\{2,3\}\}$. This difference in partitioning tasks turns out to be crucial for implementability.

In order to complete the model, assume that each department l incurs costs of $\frac{1}{2}a_i^2$ for every task i to which it is assigned, enjoys rewards r^l, is risk-neutral and that its preferences can be represented with the utility function: $u^l(w,a) = r^l - \frac{1}{2}a_i^2$.

Before turning to implementability of generic activity choice vectors, let us examine when a specific vector $\mathring{a} = (\mathring{a}_1, \mathring{a}_2, \mathring{a}_3)$ can be induced using rewards to agent l, which are linear in the signal: $r^l(s_1,s_2) = r_0^l + r_1^l s_1 + r_2^l s_2$.[9] Given linear contracts, both departments' utility functions are concave in their choices, so that first-order conditions are necessary and sufficient for their optimal choices. Consider the task partition $\{\{1,2\},\{3\}\}$. Then, the production department's first-order conditions are the following:

$$(r_1^P, r_2^P) D_{(a_1,a_2)}\mu(a) = D_{(a_1,a_2)}c(a),$$

where the entry in column i and row j of matrix $D_{(a_1,a_2)}\mu(a)$ stands for the marginal effect of the i-th task on the j-th signal, and $D_{(a_1,a_2)}c(a) = (\frac{\partial c}{\partial a_1}, \frac{\partial c}{\partial a_2})$ are the marginal costs of programming and debugging. In other words, the

[9] Corollary 1 later ensures that imposing linearity is without loss of generality.

production department can be induced to choose activity $\mathring{a}^P = (\mathring{a}_1, \mathring{a}_2)$, if and only if some linear combination of the marginal effect of activity choices on signals, $D_{(a_1,a_2)}\mu$, equals that on costs, $D_{(a_1,a_2)}c$. Similar characterizations have been put forward by Feltham and Xie (1994) and Corts (2007).[10] Observe that the two marginal effects that appear in the characterization reflect two very different dimensions: changes in the signal parameter concern the information technology, while changes in costs come from agents' preferences. One may suspect that this observation is due to linear signals, contracts, and other specific assumptions imposed in the example (as well as in the literature). The general analysis later shows, however, that all that is required is a certain separability of the utility and signal functions.

Now turn to any generic activity $a = (a_1, a_2, a_3)$ and suppose that debugging is located in the production department $\{\{1,2\},\{3\}\}$. Inserting the marginal effects in the necessary conditions yields:

$$(r_1^P, r_2^P)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = (a_1, a_2) \quad \text{or} \quad a_1 = r_1^P + r_2^P = a_2.$$

Hence, programming and debugging effort must be the same for an activity choice vector to be implemented; generic activity choices cannot be induced (the set $\{a | a_1 = a_2\}$ is not dense in $\mathbb{R}^2$). If there are benefits from focusing on programming rather than debugging, e.g., because better programmed software is easier to maintain or adapt at a later stage, these benefits cannot be reaped.

[10]Feltham and Xie (1994) observe that an activity choice is implementable if and only if 'it is spanned by the set of performance measure coefficients'—see their Appendix B. Corts (2007) notes finds that optimal linear contracts induce first-best efforts whenever signals 'span the [agent's] set of tasks' (see his Proposition 6).

Alternatively, consider assigning debugging to the marketing department: $\{\{1\},\{2,3\}\}$. Then, the first-order condition for the production department is $r_1^1 = a_1$ and there is no restriction on which programming effort can be induced. For the marketing department, an activity choice can be implemented if (and only if) it meets the following conditions:

$$(r_1^M, r_2^M)\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = (a_2, a_3) \quad \text{or} \quad a_2 = r_1^M + r_2^M \text{ and } r_1^M = a_3.$$

An arbitrary activity vector (a_2, a_3) can be induced by setting $r_1^M = a_3$ and $r_2^M = a_2 - a_3$. Hence, any generic activity $a = (a_1, a_2, a_3)$ can be induced when the marketing department decides on debugging. If there are benefits from focusing on programming, they can be realized with this task assignment and agency costs are lower.

Why can generic activity choices only be induced when debugging is assigned to the marketing but not to the production department? One feature that distinguishes the two task assignments is the departments' leeway to generate observable signals. When the production department carries out debugging, it has several ways to generate the same distribution of signals. For example, given that the marketing department chooses $a_3 = 1$, the production department can produce the parameter vector $(\mu_1, \mu_2) = (2,3) = (a_1 + a_2, a_1 + a_2 + 1)$ either by focusing on programming $(a_1, a_2) = (2, 0)$ or by engaging in both programming and debugging $(a_1, a_2) = (1, 1)$. In contrast, departments have no choice on how to generate signals when the marketing department carries out debugging. Their activity choices are *(conditionally) identified*. Given (μ_1, μ_2) and (a_2, a_3), the production department's choice must be $a_1 = \mu_1 - a_2$. Conversely, given (μ_1, μ_2) and a_1, the marketing department's choices have to be $a_2 = \mu_1 - a_1$ and $a_3 = \mu_2 - \mu_1$. How conditional identification relates to seemingly similar notions in the literature on partnerships as well as to the concept of sufficient statistics is discussed after it is properly defined in Section 3.

Intuitively, conditional identification and implementability are linked because it is easier to provide incentives for an activity if this activity can be inferred from signals. Formally, they are related by the rank of agent l's marginal effect matrix, $D_{a^l}\mu(a)$. On the one hand, agent l's activity choice, a^l, can be inferred from $\mu = (\mu_1, \mu_2)$ if and only if μ is invertible in a^l, which (by the linearity of μ) is the case whenever the marginal effect matrix, $D_{a^l}\mu(a)$, has full rank. On the other hand, a generic activity choice vector a can be induced if and only if there is a linear combination (r_1^l, r_2^l) of the columns of $D_{a^l}\mu(a)$ that equal marginal costs, which again holds whenever $D_{a^l}\mu(a)$ has full rank. Appropriately adjusted, the relationship between identifiability and 'signal independence' (as measured by the rank of the marginal effect matrix) generalizes to signals that are not linear in activity choices (Proposition 2). The respective result can then be used to establish the fundamental link between implementing generic activities and conditional identification (Theorem 1).

The insights from this section on implementability and identification do not rely on the rather specific assumptions made; they are valid for a more general framework which is presented in the following section.

3 General Framework

This section introduces a general framework and respective definitions to discuss implementability and identification.

Activity and partition. Take a generic non-contractible activity that should be implemented for some exogenous reason. The activity involves n relevant dimensions,[11] $i \in N := \{1, \ldots, n\}$, for each of which there is a choice to be made $a = (a_1, \ldots, a_n)$, where a is from some open and convex set $A \subseteq \mathbb{R}^n$. As customary, dimensions are called tasks. In the software example, the activity has three dimensions and the set of tasks is $N = \{1, 2, 3\}$.

Let $\mathcal{P}$ be some partition (disjoint decomposition) of the set of tasks N and label the elements of $\mathcal{P}$ by $l = 1, \ldots, m$. These elements, $N^l \in \mathcal{P}$, represent the set of tasks for which the choices $\{a_i\}_{i \in N^l}$ are determined by the same agent l. For example with the finest partition $\mathcal{P}_0 := \{\{1\}, \{2\}, \{3\}\}$, there are three agents, where the first programs, the second debugs, and the third markets the software. Denote agent l's choice for each of his tasks, $i = 1, \ldots, |N^l|$, by the $1 \times |N^l|$ vector a^l. For example, the activity choice vector by the marketing department given partition $\{\{1\}, \{2, 3\}\}$ would be $a^M = (a_2, a_3)$. Observe that tasks can always be re-labeled such that the first $|N^l|$ tasks belong to agent l: $a = (a^l, a^{-l})$, where a^{-l} are the choices by other agents.

Agents' utility. Agent l incurs (strictly convex) costs $c^l(a)$ from activity a: $c^l : A \to \mathbb{R}$. On the other hand, the agent can be rewarded in some form, which for simplicity's sake, is taken to be monetary. Denote any rewards to agent l by r^l. Overall, agent l's utility strictly increases in rewards r^l and decreases in costs: $u^l(r^l, c^l)$. Agent l's utility outside the relationship with

[11] Dimensions that are irrelevant, e.g., because they do not affect the mechanism designer's utility are excluded.

the principal is denoted by $\underline{u}^l$.

The functional form of the utility allows for the effect of overall costs to change with wealth, which is typically assumed away in moral-hazard models.[12] However, it implies that rewards and costs exhibit a certain separability. In particular, costs at task i cannot change in relation to those at task i' when the agent becomes poorer or richer.

In order to ensure that agent l's problem has an inner solution that can be found using standard calculus, assume that u^l is differentiable and jointly concave in r^l and c^l. Since any desired marginal effect of the activity on the utility can be modeled by adjusting c^l, it is without loss of generality to assume that the marginal effect of costs on utility is bounded: $0 > \frac{\partial u^l}{\partial c^l} \geq -1$.

The framework allows costs, number, and identity of agents to change with the task partition. While any activity choice can trivially be implemented by finding an agent who likes that choice; this method does not work generically—unless agents are indifferent with respect to their behavior, which is ruled out by the assumption that costs be strictly convex.

Signal structure. The signal structure follows the tradition of Spence and Zeckhauser (1971) as well as Ross (1973) in using a state-space representation. Each signal j, with $j = 1, \ldots, k$, is a (non-constant) function of a one-dimensional parameter μ_j that represents the systematic effect of agents' activity and a continuously distributed real-valued random variable ϵ_j that

[12] For exceptions, see Mookherjee (1997), Thiele and Wambach (1999), Schnedler (2011).

is beyond agents' control:

$$\begin{aligned} S_j : \mathbb{R}^2 &\rightarrow \mathbb{R} \\ (\mu_j, e_j) &\mapsto S_j(\mu_j, e_j), \end{aligned}$$

where e_j is a realization of ϵ_j. The parameter μ_j itself is a function of the activity choice vector a:

$$\begin{aligned} \mu_j : \mathbb{R}^n &\rightarrow \mathbb{R} \\ a &\mapsto \mu_j(a). \end{aligned}$$

The functional form of signals requires that the effect under control of agents, μ_j, can be separated from the effect beyond their control, ϵ_j. This assumption is, of course, no restriction as long as the activity itself is one-dimensional ($n = 1$) as in Spence and Zeckhauser (1971) and Ross (1973); respective models can simply be embedded by defining $\mu(a) = a$ for all $a \in \mathbb{R}$. In most moral-hazard models with multi-dimensional activities ($n > 1$), the assumption is fulfilled because there is no interaction between randomness and individual task choices. An interesting exception is the specific knowledge model by Raith (2008), where the *relative* marginal effect of choices at two tasks varies randomly.

For state-space representations, signals are typically assumed to be concave in activity choices for any state (see e.g. Conlon, 2009). In line with this assumption, let S_j be concave in μ_j for almost all e_j and μ_j be concave in a, so that S_j is concave in a.

The random variables $S_j(\mu_j, \epsilon_j)$ may be continuously distributed with density, $f_j(s_j, \mu_j)$, discretely distributed with probability function, $p_j(s_j, \mu_j)$, or some mixture of both.[13] In order to use marginal arguments later, assume that density and probability function are differentiable in μ_j and that the support of S_j is constant in μ_j. Moreover, suppose that $S_j(\mu_j, \epsilon_j)$ stochastically increases in μ_j, which is not too restrictive given that the relationship between μ_j may increase or decrease in the choice at task i, a_i.

Incentives. Agents' utility and the signal structure are assumed to be common knowledge. The mechanism designer can influence agents' behavior by tying rewards to signals—either formally or in a self-enforcing manner. For simplicity, assume that signals are verifiable and r^l is a real-valued (measurable) function $r^l : \mathbb{R}^k \to \mathbb{R}$, that assigns a reward to agent l, $r^l(s)$, for each realization $s = (s_1, \dots, s_k)$ of the random vector $S = (S_1, \dots, S_k)$.[14] For example, rewards may be *linear* in realizations: $r^l(s) = r^l_0 + r^l_1 s_1 + \dots + r^l_k s_k$. Another example would be bonuses that are paid when a certain threshold is reached.

Derivatives. In order to summarize agents' marginal effects, the following notation is used. $D_a\mu(a)$ stands for the matrix with a row for each of the k signals and a column for each of the n tasks; the (j, i)-th entry of this matrix describes the effect of the i-th task in the activity vector, a_i, on the j-th signal, S_j given a: $\frac{\partial S_j}{\partial a_i}$. $D_{a^l}\mu(a)$ is the respective marginal effect matrix for the activity choice vector under control of agent l, a^l. $D_a c^l(a)$ is a row

[13] Since any distribution can be decomposed into an absolutely continuous, discrete and singular measure by Lebesgue's decomposition theorem, the assumption essentially means that there is no singular component.

[14] Alternatively, the framework could be formulated in a repeated game setting similar to Fudenberg, Levine, and Maskin (1994).

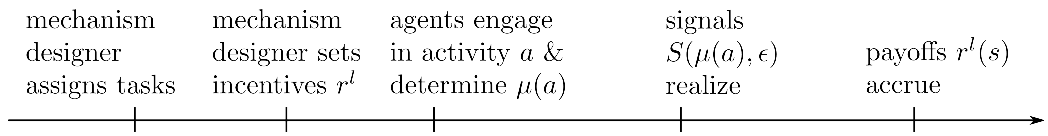

Figure 1: Sequence of Events.

vector with an entry for each of the n tasks, where the i-th entry describes the effect of the i-th task on costs: $\frac{\partial c^l}{\partial a_i}$ and $D_{a^l} c^l(a)$ the respective row vector for the $|N^l|$ tasks under control by agent l.

Figure 1 gives an overview of the main elements of the model and the sequence of events. After task assignment, the structure is that of a standard moral hazard problem with one or many agents—depending on the task assignment. Once a contract is signed, agents $l = 1, \ldots, m$ play a game, the payoffs of which result from rewards being attached to signals. The payoffs hence only depend on activity choices via $\mu(a)$ and each agent's $c^l(a)$ but not on the order of moves by the agents. An activity choice vector is implementable if it forms a Nash-equilibrium in the agents' game.

Definition 1 (Implementability)**.** *Given a signal structure $\mathcal{S}$ and a partition $\mathcal{P}$, a specific activity choice $\mathring{a} = (\mathring{a}_1, \ldots, \mathring{a}_n)$ is implementable if there are reward functions r^l for each agent l such that the activity is a Nash equilibrium. Let $\mathcal{I}$ be the (possibly empty) set of all implementable activity choices. If $\mathcal{I}$ is dense in A, activity choices are generically implementable.*

Although inducing an activity is not an inference problem, one can ask whether it is possible to recover activity choices from the signal distribution. As signals stochastically increase in the parameter vector, $\mu(a)$, the latter uniquely determines the distribution and can be inferred from sufficiently many (possibly infinitely many) signal realizations. The question then boils down to whether activity choices can be deduced from the parameter vector $\mu(a)$.

Definition 2 (Identification)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, the activity of agent l is (conditionally) identified if for any activity choices by other agents, a^{-l}, two distinct choices by agent l lead to different signal distributions:*

$$\text{for all } a^{-l} \text{ and } a^l \neq \tilde{a}^l : \quad \mu(a^l, a^{-l}) \neq \mu(\tilde{a}^l, a^{-l}).$$

Agents' activity is said to be identified given $\mathcal{S}$ and $\mathcal{P}$, if it is identified given $\mathcal{S}$ and $\mathcal{P}$ for all agents $l = 1, \ldots, m$. Otherwise, there is an identification problem.

Identification imposes no restrictions on the interaction between different tasks—neither in the agents' cost function nor in the production of signals. All that matters for identification is that there is an injective mapping from each agent's decision a^l to μ conditional on a^{-l}. This distinguishes the concept from similar concepts in the literature on partnerships—e.g., the concept of pairwise identifiability by Fudenberg, Levine, and Maskin (1994). In this literature, it is important to identify one agent who has not deviated from some efficient activity choice, say a^*, (see e.g. Battaglini, 2006) and who can thus act as a budget breaker by receiving any fines imposed on deviators.

Activities may well be conditionally identified while it is impossible to find out who deviated. Reconsider the software example from Section 2 with partition $\{\{1\}, \{2, 3\}\}$ and suppose that the efficient activity is $a^* = (1, 1, 1)$ and that the production department deviates by reducing programming effort $a = (0, 1, 1)$. Given the deviation signal parameters are $\mu(0, 1, 1) =$

$(a_1 + a_2, a_1 + a_2 + 1_3) = (1, 2)$. The same parameter values, however, may also be generated by a deviation of the marketing department with respect to debugging effort: $a = (1, 0, 1)$. It is thus not possible to identify who deviated given parameter values $(1, 2)$. At the same time, agents' activity choices are conditionally identified—as seen in Section 2. Still, conditional identification is not a weaker concept. In partnership models, only a deviation from a specific activity choice vector matters (the efficient one), whereas here identification concerns the general capability to (conditionally) infer activity choices from parameters.

Holmström (1979) famously suggested to assess the value of an additional signal S_2 by checking whether the original signal S_1 is a sufficient statistic for activity a. An additional signal S_2 that is informative in the sufficient statistic sense, however, does not necessarily help with conditionally identifying activity choices. Consider a single agent who faces two tasks and two signals, $S_j(\mu_j, \epsilon_j) = \mu_j + \epsilon_j$ with $\mu_1(a) = \mu_2(a) = a_1 + a_2$ and let ϵ_j be an independently standard normally distributed error term. The agent's activity cannot be identified—irrespective of whether S_1 or both signals are used. On the other hand, signal S_2 conditional on S_1 is normally distributed with mean $a_1 + a_2$ so that S_1 is no sufficient statistic for S_2 because its distribution depends on activity choices. Signal S_2 is hence informative about the activity in Holmström's sense but does not help with identification.[15]

[15]Observe that Holmström's well-known sufficient statistic result is restricted to distributions from the exponential class with rank one (Amershi and Hughes, 1989) and can fail if activities are multi-dimensional (Holmström and Milgrom, 1991) or if income affects marginal costs of effort (Schnedler, 2010). In contrast, results on identification derived here are independent of the specific distribution, hold for single- and multi-dimensional activities, and allow for interaction between effort and wealth.

4 Implementability and Identification

The central aim of this section is to prove that implementability is limited if and only if agents' activity is not identified. For this proof, it is helpful to first characterize which activities are implementable (Proposition 1). Given this characterization, limited implementability can be traced back to the rank of the marginal effect matrix $D_{a^l}\mu(a)$ (Corollary 2). On the other hand, identification problems are also related to this rank (Proposition 2). Combining both results yields that implementability is restricted if and only if there is an identification problem.

Given some partition, signal structure and incentives, each agent l chooses activity vector $\mathring{a}^l$ given the behavior of others, say $\mathring{a}^{-l}$, such that it maximizes his expected utility:

$$\mathring{a}^l \in \text{argmax}_{a^l \in A} \mathrm{E}_\epsilon \left[u^l \left(r^l \left(S \left(\mu(a^l, \mathring{a}^{-l}), \epsilon \right) \right), c^l(a^l, \mathring{a}^{-l}) \right) \right]. \tag{1}$$

Exploiting that signal densities (or probability functions) are differentiable in the parameter μ_j, the expected utility can be differentiated in the activity vector—even if the reward function r^l is not differentiable (by Lemma 1 in Appendix B). Implementing a specific activity vector $\mathring{a}$ as an inner maximizer then requires that the necessary first-order conditions hold and that the $1 \times |N^l|$ vector, which describes the derivatives of the utility with respect to agent l's choice a^l is zero evaluated at the implemented activity $\mathring{a}^l$: $\frac{d}{da^l}\mathrm{E}_\epsilon \left[u^l \left(r^l \left(S \left(\mu(a^l, \mathring{a}^{-l}), \epsilon \right) \right), c^l(a^l, \mathring{a}^{-l}) \right) \right] \big|_{a^l = \mathring{a}^l} = (0, \ldots, 0)'$.

Recall that activities affect signals and utility only via μ and c^l. As a result, it becomes possible to separate marginal gains from losses and further decompose both terms. Marginal gains occur because the activity vector affects the signal parameters, $D_{a^l}\mu(a)$, which in turn influence the expected utility, $D_\mu \mathrm{E}_\epsilon \left[u^l(r^l(S(\mu,\epsilon)), c^l)\right]$. Marginal losses unfold because the activity vector affects costs $D_{a^l} c^l(a)$, which then decreases expected utility, $\frac{d}{dc^l}\mathrm{E}_\epsilon \left[u^l(r^l(S(\mu,\epsilon)), c^l)\right]$. Using this decomposition, the necessary conditions for implementing agent l's activity vector $\mathring{a}^l$ given $\mathring{a}^{-l}$ are:

$$\begin{aligned} &D_\mu \mathrm{E}_\epsilon \left[u^l(r^l(S(\mu,\epsilon)), c^l)\right] \cdot D_{a^l}\mu(a)\big|_{a=\mathring{a}} \\ &= -\frac{d}{dc^l}\mathrm{E}_\epsilon \left[u^l(r^l(S(\mu,\epsilon)), c^l)\right] \cdot D_{a^l} c^l(a)\Big|_{a=\mathring{a}} . \end{aligned} \tag{2}$$

The necessary conditions (2) must hold for all agents at activity choice $\mathring{a}$ for this choice to be implementable. Dividing by the marginal effect of costs on the expected utility $\frac{d}{dc^l}\mathrm{E}_\epsilon \left[u^l(r^l(S(\mu,\epsilon)), c^l)\right]$ yields that a vector $\mathring{a}$ is only implementable if for each agent some linear combination of his activities' marginal effect on signals equals that on costs (a formal proof is given in Lemma 2 in Appendix B). The advantage of this condition is that it is independent of the incentive scheme. Since it is derived from the first-order conditions, it is necessary for implementability. More interestingly, the condition is also sufficient. Once the condition is met, it is possible to find rewards that implement the desired activity.

Proposition 1 (Implementable activities)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, activity choice $\mathring{a}$ can be implemented if and only if for each agent, some linear combination of his activities' marginal effect on signals equal that on costs given $\mathring{a}$:*

$$\mathring{a} \in \mathcal{I} \textit{ iff for all } l \textit{ there is some } \lambda^l \in \mathbb{R}^k \textit{ with } \lambda^l\, D_{a^l}\mu(a)\big|_{a=\mathring{a}} = D_{a^l} c^l(a)\Big|_{a=\mathring{a}} . \tag{3}$$

The idea of the proof is the following (the formal proof can be found with all other proofs in Appendix A). If the desired activity is implementable and hence a Nash equilibrium, this choice must be optimal for each agent l, which directly implies condition (3) (see Lemma 2 in Appendix B). The proof that the condition is also sufficient is more involved. The intuition is the following. Suppose that all agents other than agent l engage in the desired activity and consider a linear reward scheme for this agent: $r^l(s) = r_0^l + r_1^l s_1 + \ldots + r_k^l s_k$. With two assumptions from our model, which are considered 'completely standard' by Conlon (2009), namely that signals be concave in activities and agents' utility be increasing and concave in rewards, linearity of rewards implies that the first-order conditions are also sufficient. The difficulty then lies in showing that given condition (3), one can find weights r_j^l in the linear reward scheme such that the first-order conditions are met for this scheme at the desired choice $\mathring{a}^l$. The difficulty arises from the fact that adjusting weights may affect agent l's marginal utility as he becomes richer (or poorer). Potentially, agent l's marginal utility from rewards may even become negligible. This can be avoided by selecting a base wage r_0 in dependence of r_j^l such that the agent's participation constraint binds. Then, his expected utility is constant and his marginal utility from rewards is bounded away from zero (see Lemma 3). After having established that condition (3) allows us to find weights in a linear scheme for which the first-order conditions hold (at the desired activity) and that these conditions are also sufficient, it follows that agent l has no incentive to deviate from the desired activity as long as no other agent does so. Since the argument applies to an arbitrary agent l, it is possible to find a respective (linear) reward scheme for any agent such that the desired activity becomes a Nash equilibrium.

Proposition 1 offers a general characterization of implementable activities for a given task partition amongst several agents. It emphasizes the fundamentals that matter for implementability: the marginal effects of activities on signals, $D_{a^l}\mu(a)$, and costs, $D_{a^l}c^l(a)$. Apart from the assumption that the agents' problems be concave, very little restrictions are placed on $\mu(a)$ and $c^l(a)$. It is noteworthy that condition (3) is otherwise independent of variables that are usually considered important like the agents' degree of risk aversion or the substitutability of effort in signals or costs.

The characterization of implementable activities (3) closely relates to first-order conditions. Often first-order conditions are used to replace the incentive constraints when finding optimal incentives. This first-order approach has been the object of a debate for nearly four decades.[16] In view of this debate, it should be stressed that the analysis underpinning Proposition 1 and hence the subsequent results, does not rely on this approach. Implementable activities can be and are characterized here without determining optimal incentives; consequently, it is not necessary to employ the first-order approach. Although the aim here is implementability and not optimality, results can be used to better understand second-best contracts and assignments—for examples, see Section 6.3 or 6.5.

Since the proof uses linear rewards to show that condition (3) suffices for implementation, we immediately obtain the following corollary.

Corollary 1. *Any activity $\mathring{a}$ that can be implemented can also be implemented with linear rewards.*

The corollary shows that imposing linearity when trying to find out which activities are implementable is without loss of generality. It thus justifies

[16]This debate started with the insightful work by Mirrlees (1974) and does probably not end with the illuminating analysis by Conlon (2009).

the approach by Feltham and Xie (1994) and Corts (2007) of restricting attention to linear rewards when examining implementation. This contrasts with the highly restrictive nature of linear rewards when determining optimal contracts (see Mirrlees, 1974).

Proposition 1 is helpful from a technical point of view because it reduces the question of implementability to that of the existence of a solution to a linear equation system. This means that standard results from linear algebra can be used to check for implementability. Take, for example, the result that a linear equation system has a solution if the rank of the extended coefficient matrix is equal to the rank of the coefficient matrix itself. Applying this result to Proposition 1 yields the following corollary.

Corollary 2 (Rank characterization for implementable activities)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, activity $\mathring{a}$ is implementable if and only if for all agents, marginal costs are in the row-space of the matrix describing their marginal effects at $\mathring{a}$ on signals:*

$$\textit{For all } l: \ \ \textit{rank}\,(D_{a^l}\mu(a))\Big|_{a=\mathring{a}} = \textit{rank}\begin{pmatrix} D_{a^l}\mu(a) \\ D_{a^l}c^l(a) \end{pmatrix}\Bigg|_{a=\mathring{a}}. \tag{4}$$

This characterization of implementable activities is useful because it expresses implementability using the rank of the marginal effect matrix, $D_{a^l}\mu(a)$. Fudenberg, Levine, and Maskin (1994) provide a necessary condition for implementability that is also based on ranks albeit in a different setting. Their individual rank condition follows from keeping an agent in a repeated game indifferent between choosing a desired activity and deviating. As signal realizations and the set of choices are finite, distributions induced by a specific choice can be described by a vector of probabilities. Individual full rank means that the probability vectors for different activity choices differ. There is hence a direct link between the rank of the matrix containing the probability distributions for different choices and identification. The following result asserts that a similar link can be established when the activity space is continuous so that the matrix describes marginal effects on signals rather than probabilities.

Proposition 2 (Identification and signal independence)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, the activity is identified for all agents if and only if all agents face at least as many independent signals as tasks:*

$$\textit{For all } l \textit{ and } \mathring{a}: \; \textit{rank}(D_{a^l}\mu(a))\,\big|_{a=\mathring{a}} \geq |N^l|.$$

The intuition for the result is the following. Suppose there are less independent signals than tasks for some agent or more formally that the rank of the marginal effect matrix is not large enough for some vector $\mathring{a}$. Then, there is a direction in which a change of some agent's decision does not affect the parameter vector. Choices along this direction can hence not be identified and the mechanism designer faces an identification problem. Conversely, having sufficiently many independent signals, i.e., a sufficiently large rank of the marginal effect matrix, means that any direction leads to a different parameter vector, so that the agent's activity affects parameter values and is identified.[17]

The proposition clarifies in what sense the richness of the same signal structure $\mathcal{S}$ can vary. It may or may not be conditionally identified, depending on the task assignment. The proposition can also be re-interpreted as a

[17]The rank condition is somewhat weaker than requiring invertibility of the agents' marginal effect matrix. Invertibility would in addition mean that the matrix has to be quadratic, i.e., the number of signals would have to be the same as the number of tasks.

characterization of task assignments that are free from identification problems. In the next section, this interpretation is used to derive necessary and sufficient conditions for task assignments to solve identification problems.

Using that implementation and identification depend on the rank of the same matrix yields the following central result.

Theorem 1 (Implementability and identification)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, any activity is implementable if and only if (all) agents' activity is identified.*

The logic of the proof is the following. Whenever there is no identification problem, there are at least as many independent signals as tasks for all agents and activities by Proposition 2. The central step in the proof is to show that this is fulfilled whenever the rank condition (4) from Corollary 2 holds.

The proof is indirectly based on Proposition 1, which uses linear rewards. Linear rewards are problematic in the canonical hidden-action model with limited liability and a risk-neutral agent, i.e., $u^l(r^l, c^l) = r^l - c^l$ with the additional restriction $r^l \geq 0$, because linearity implies that the agent may lose money for some (possibly rare) signal realizations.[18] Still, identification also leads to implementability in this setting. Since activities are identified, there are signal realizations which are most likely when each agent selects the desired choice; tying a finite bonus to these realizations then induces the desired activity (see Proposition 5 in Appendix C). The message of the theorem thus also holds in a somewhat different framework.[19]

[18]For a textbook treatment see, for example, Macho-Stadler and Perez-Castrillo (1997).

[19]The proof employs direct utility comparisons rather than first-order conditions and hence also works for finite sets of activity choices. On the other hand, it requires risk-neutral agents.

The theorem pins down the signal imperfection that hampers implementability: the lack of identification. Interestingly, it is only the systematic effect of the activity on signals that matters and not the distribution of the error terms. The theorem also formally establishes how task assignment can affect implementability: by (conditionally) identifying activity choices.

The theorem may suggest that restricted implementability is essentially a problem of inferring activities from signal realizations similar to identification problems in econometrics. The analogy, however, is limited. If the activity of agent l is identified according to Definition 2, an econometrician still has to know the choices of the other other agents, a^{-l}, and to observe a sufficient number realizations s of the signal vector S (possibly infinitely many) before activity vector a^l can be deduced. In contrast, knowing the choices a^{-l} is not required and a single realization s of S suffices in order to induce generic activity choices. The reason is that conditioning rewards on this realization provides the correct marginal gains and hence ex-ante incentives for agents to choose the activity a. Given these incentives, the mechanism designer knows that agents behave in the desired manner and there is no need to deduce activity choices from signals.

5 Identification by Task Assignment

The previous section has shown that implementability is limited whenever there are identification problems. Moreover, Proposition 2 introduced a simple characterization for task assignments that are free of such problems. Building on this proposition, the present section provides general conditions under which task assignment can overcome identification problems (Corollary 3 and 4) and more specific conditions on the respective number of agents, independent signals, and tasks (Proposition 3 and 4). Before linking multi-tasking to the question when task assignment can overcome limited implementability, let us single out a specific class of identification problems.

Definition 3 (Identification problem caused by multi-tasking)**.** *Suppose there is an identification problem given signal structure $\mathcal{S}$ and some partition $\mathcal{P}$. Then, this identification problem is caused by multi-tasking if there is no identification problem given signal structure $\mathcal{S}$ and the finest partition, in which no agent carries out more than one task:* $\{\{1\}, \ldots, \{n\}\}$.

As an example for an identification problem caused by multi-tasking, return to Section 2. Assigning debugging and programming to the production department led to an identification problem. If programming, debugging, and marketing are be carried out by three different units, this identification problem disappears. It is hence caused by multi-tasking. Multi-tasking can thus cause identification problems and thereby restrict the set of implementable activity choices and increase agency costs. This type of multi-tasking problem is very distinct from that examined by Holmström and Milgrom (1991)—an issue to which we return in Section 6.6.

If multi-tasking causes an identification problem, this problem can by definition be solved through task assignment—simply, by assigning each task to a different agent. Conversely, an identification problem that can be solved through some task assignment can also be alleviated with the finest partition. The intuitive reason is that the finest partition leaves agents less scope to generate signals than any other partition (a formal proof can be found in Appendix A). The following corollary sums up the argument.

Corollary 3 (Identification problems, task assignment, and multi-tasking). *Given signal structure $\mathcal{S}$, identification problems can be solved by some task assignment $\mathcal{P}$ if and only if they are caused by multi-tasking.*

With this corollary, it becomes simple to determine whether identification by task assignment is possible: all that needs to be checked is whether the finest partition leads to an identification problem or not. If the finest partition is used and each task is carried out by a different agent, identification boils down to the parameter vector being affected whenever the agent at task i changes his choice a_i, while the other agents' behavior is fixed. This observation allows us to derive a simple characterization of signal structures for which task assignment can overcome identification problems.

Corollary 4 (Identification by task assignment). *Given signal structure $\mathcal{S}$, identification problems can be avoided by some task assignment $\mathcal{P}$ if and only if different choices at any task i lead to different signal distributions (holding constant the choices at all other tasks), or formally,*

$$\text{for all } a = (a_1, \ldots, a_n), i \in \{1, \ldots, n\}, \text{ and } \tilde{a}_i \text{ with } \tilde{a}_i \neq a_i :$$
$$\mu(a_1, \ldots, \tilde{a}_i, \ldots, a_n) \neq \mu(a_1, \ldots, a_i, \ldots, a_n).$$

Re-phrasing the corollary, task assignment is a viable identification strategy if the parameter vector is an injective function of each decision a_i for given constant choices at all other tasks. Notice that conditional injectivity is sufficient. In particular, the parameter vector, μ, does not have to be injective in the whole activity vector, a. As seen in the software example, it may be impossible to infer behavior from signals and still task assignment overcomes the identification problem.

The task assignment that avoids identification problems may be expensive in the sense that it requires a lot of personnel. How many agents are necessary? How many are sufficient? The remainder of the section attempts to answer these questions for a signal structure that in principle allows for identification, i.e., behavior would be identified with the finest partition; there are simply not enough agents.

Proposition 3 (Necessary condition)**.** *Given signal structure $\mathcal{S}$, identification problems can only be alleviated by partitions for which the average number of tasks per agent is at most the overall number of independent signals $\tilde{k}$:*

$$\frac{n}{m} \leq \tilde{k},$$

where $\tilde{k} := \min_a rank(D_a \mu(a))$.

The number of independent signals *for some agent l* can at most be the overall number of independent signals, $\tilde{k}$. If an agent is assigned more tasks than the overall number of independent signals, this leads to an identification problem (by Proposition 2). The maximal number of tasks that can possibly be assigned to each agent is thus $\tilde{k}$. With m agents, the maximal possible number of tasks, n, such that agents' behavior is still identified (and implementability is ensured) is hence $m \cdot \tilde{k}$.

Corts (2007) observes that the number of signals must be at least as large as the average number of tasks: $k \geq n/m$.[20] Since there are always at least as

[20] Corts' condition originally concerns the question when the optimal linear contract implements the first-best with risk-neutral agents under generic production functions and not when task assignment identifies activities. However, the rank condition in his Proposition 6 is linked to identification by Proposition 2 in the present paper.

many signals as independent signals, $k \geq \tilde{k}$, the proposition tightens Corts' boundary.

The condition in the proposition resembles that of Battaglini (2006) for implementing efficient production in partnerships. While Battaglini's condition is sufficient for his problem, the condition is not sufficient in the present context. As an example, take two agents and the following marginal effect matrix, which could be seen as an extension of the software problem with an additional activity choice a_4 that improves stability:

$$D_a(\mu(a)) = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

There are two independent signals, i.e., the minimum rank of the matrix is two, $\tilde{k} = 2$, because the two signal coefficient vectors in the rows are linearly independent. Given the two departments, the average number of tasks per department is likewise two, $n/m = 4/2 = 2$ and the condition in the proposition is met: $\tilde{k} = 2 = 4/2 = n/m$. Still, the identification problem cannot be eliminated by any partition among the two departments. Take the three tasks affecting stability: $\{1, 2, 4\}$. However these tasks are assigned, at least two of them have to be carried out by the same department l. But for this department, signals are not independent, $\text{rank}(D_{a^l}\mu(a)) = 1$. It thus faces more tasks than independent signals and there is an identification problem (by Proposition 2). In summary, the condition that the average number of tasks per agent must be larger than the total number of independent signals is necessary but not sufficient. A sufficient condition is given in the next result.

Proposition 4 (Sufficient condition)**.** *Given signal structure $\mathcal{S}$ and an identification problem that is caused by multi-tasking, the identification problem* *can be alleviated by some partition $\mathcal{P}$ if the number of agents and total number of independent signals exceeds the number of tasks*

$$m + \tilde{k} > n,$$

where $\tilde{k} := \min_a \mathit{rank}(D_a \mu(a))$.

The intuition behind this result is the following. With a total number of $\tilde{k}$ independent signals, $\tilde{k}$ tasks can be carried out by one agent without causing an identification problem. To guarantee that the remaining $n - \tilde{k}$ tasks are not resulting in such a problem, they have to be assigned to $n - \tilde{k}$ agents. Thus, $n - \tilde{k} + 1$ agents are sufficient to avoid the identification problem.

The boundary is tight: with one agent less, i.e., $m = n - \tilde{k}$, it is no longer certain that identification can be achieved. We have just seen an example with two agents, four tasks and an overall number of two independent signals, so that $m = n - \tilde{k}$, in which no partition conditionally identified behavior.

The condition, $m > n - \tilde{k}$, is sufficient but not necessary. Take two agents, $m = 2$, and the following marginal effect matrix:

$$D_a(\mu(a)) = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

The two signals are hence independent, $k = \tilde{k} = 2$. Accordingly $m + \tilde{k} = 2 + 2 = 4$, and thus $m + \tilde{k} = 2 + 2 = 4 = n$ and the condition is violated. With partition $\{\{1, 2\}, \{3, 4\}\}$, each agent has two independent signals and two tasks. Hence, activities are identified (by Proposition 2) although $m = n - \tilde{k}$.

6 Applications

This part exploits the generality of the framework and applies the results to established and new moral-hazard models. Section 6.1 revisits models that feature improved implementability as a result of task assignment and explains why assigning tasks differently leads to identification (and hence unlimited implementability) in these models. Section 6.2 illustrates with a short example that a shift in accountability is not essential for augmented implementability. Section 6.3 explains optimal task bundling by a trade-off between gains from more control (augmented implementability) and cost complementarities. Section 6.4 returns to the early moral-hazard literature and points out that implementability is typically unrestricted under its assumptions. Section 6.5 presents a single-task model with an identification problem and shows how biased signals can overcome this problem. Section 6.6 argues that Holmstrom and Milgrom's famous multi-tasking article features no implementability problems due to multi-tasking. Finally, Section 6.7 takes a common assumption from the recent multi-tasking literature (linearity of signals) and shows that under this assumption, task assignment can generically overcome implementability problems.

6.1 Advocates, Specialists, and Joint Accountability

Three contributions that particularly emphasize the beneficial effects of task assignment on implementation are the advocates model by Dewatripont and Tirole (1999), the specialization model by Ratto and Schnedler (2008), and the accountability model by Corts (2007). While these models differ in various dimensions (e.g., number and character of tasks, number of signals, flexibility of number of agents), the principle that underpins the result is the same: a different task assignment leads to identification.

All contributions start out with an organization structure (single investigator, single worker, individual accountability) that is plagued by an identification problem, which implies limited implementability. Moreover, signals are injective in activity choices at all tasks, so that these identification problems are caused by multi-tasking and can be overcome by task assignment (by Corollaries 3and 4). The proposed alternative task assignments (advocacy, specialization, joint accountability) exploit this opportunity by increasing the number of independent signals relative to the number of tasks, so that activity choices become identified (by Proposition 2), implementability is no longer limited (by Theorem 1), and agency costs are lower.

6.2 Identification Without Shifts in Accountability

Identification in the advocates, specialization and accountability model involves moving from a situation in which agents' payoffs are only affected by their own action (individual accountability) and randomness to one in which they also depend on other agents' decisions (team or joint accountability).[21] This may suggest that a shift in the type of accountability is at the heart of implementability restrictions and possibly related to identification. Moving from individual to joint accountability, however, is not crucial for identification (and hence neither for augmented implementability).

In the following example, identification by task assignment is achieved al-

[21]While joint accountability has a smack of a free-rider problem, this problem is absent from Dewatripont and Tirole (1999), Corts (2007), and the present paper, because the mechanism designer acts as a budget breaker as in Holmström (1982).

though agents influence each others payoffs before and after tasks are newly assigned. Return to the software model from Section 2, but suppose now that both agents dislike risky payments. In addition, assume that uncertainty in sales ϵ_2, is independent from the uncertainty about the program's stability, ϵ_1. As a departure point, take the assignment in which the production department debugs, $\{\{1,2\},\{3\}\}$, and which was plagued by an identification problem. Whatever is induced given this task partition can also be induced by exposing the risk-averse agents to less uncertainty if stability and sales signal are both used. The optimal contracts for both agents hence depend on both signals and thus on the other agent's behavior; agents are jointly accountable. We have seen that the identification problem disappears if the marketing department debugs $\{\{1\},\{2,3\}\}$. Again, it is optimal to pay both departments according to both signals: the production department to reduce noise and the marketing department to obtain the desired activity choices. Departments are thus jointly accountable under both partitions, but only one of them yields identification and unlimited implementability.

6.3 More Control versus Exploiting Complementarity

In the typical principal-agent setup, identification is, of course, not a goal in its own right but an important and sometimes overlooked means of minimizing agency costs as it may improve the principal's control of agents' opportunistic behavior. Some models trade-off this benefit of identification (without explicitly recognizing its origins) against insurance costs (Corts, 2007) or rents induced by minimum wages (Kragl and Schöttner, 2011). Another important reason not to separate tasks are complementarities, either for technical reasons or because agents like variety. This section illustrates how task assignment affects identification and hence agency costs in a simple example. The example is a metaphor for a more fundamental trade-off between better control on the one hand and exploiting complementarity on the other hand.

Consider a two-task variation of the software example ($n = 2$) that only involves programming and debugging (a_1, a_2). Suppose the two tasks exhibit complementarities and that the gains of complementarities only manifest if they are carried out by the same agent. Let costs amount to $c^B(a) = \frac{a_1^2 + a_2^2 - \gamma a_1 a_2}{2}$ if tasks are bundled and to $c^j(a) = \frac{a_j^2}{2}$ if each task $j = 1, 2$ is carried out by a different agent j, so that $\gamma \in [0, 2)$ describes the extent of complementarities. Standardize the departments' outside option to zero and let agents be risk-neutral to abstract from insurance issues. The only observable variable is the software's stability, $\mu(a) = a_1 + a_2$. There is a principal (she) who is affected by agents' behavior and who is not only interested in stability but benefits from a focus on programing: $u^P(a) = \mu(a) + \delta a_1 - r$, where r stands for any rewards given to agents and $\delta \geq 0$ captures the degree to which the principal prefers a focus on programming. The parameter δ thus describes the principal's desire to control agents' behavior.

The following analysis proceeds in three steps. First, an optimal contract is determined under task bundling and the generated surplus is computed. Second, the same is done for task separation. Finally, surpluses are compared to show that bundling is optimal whenever the principal wants to influence the agent's decision (large δ) and gains from complementarity are limited (small γ). In the analysis, attention is restricted to linear contracts. This is without loss of generality because the only friction in the model is that some activities may not be implementable; as seen in Corollary 1, linear contracts suffice to overcome this friction.

Task bundling, $\{\{1,2\}\}$, leads to an identification problem because the first agent faces two tasks but only one signal (by Proposition 2) and implementability is restricted (by Theorem 1). The principal's program is:

$$\max_{r_0,r_1} \quad (1+\delta)a_1 + a_2 - r_0 - r_1 \cdot (a_1 + a_2)$$

$$\text{such that} \quad (a_1, a_2) \in \arg\max_{\tilde{a}} r_0 + r_1 \cdot (\tilde{a}_1 + \tilde{a}_2) - c^B(\tilde{a}) \tag{IC}$$

$$\text{and} \quad r_0 + r_1 \cdot (a_1 + a_2) - c^B(a) = 0. \tag{PC}$$

The incentive constraint (IC) boils down to the implementability condition from Proposition 1:

$$r_1 \cdot D_{(a_1,a_2)}\mu(a) = D_{(a_1,a_2)}c^B(a) \;\text{ or }\; r_1 \cdot (1,1) = (a_1 - \frac{\gamma}{2}a_2, a_2 - \frac{\gamma}{2}a_1).$$

Eliminating r_1 in this equation system yields that only identical choices on both tasks are implementable, i.e., $a_1 = a_2$. Plugging this condition as well as the participation constraint (PC) into the maximization program yields

$$\max_{a_1} \quad (1+\delta)a_1 + a_1 - \frac{a_1^2 + a_1^2 - \gamma a_1 a_1}{2}. \tag{5}$$

From this, the optimal activity choices under task bundling, $(a_1^{\mathrm{B}}, a_2^{\mathrm{B}}$, can be computed to be $a_1^{\mathrm{B}} = a_2^{\mathrm{B}} = \frac{2+\delta}{2-\gamma}$. The respective surplus is

$$\pi^{\mathrm{B}} := \frac{1}{2}\frac{(2+\delta)^2}{(2-\gamma)}.$$

Task separation, $\{\{1\},\{2\}\}$, leads to conditionally identified activities because each agent faces as many independent signals as tasks (by Proposition 2) and implementability is unrestricted (by Theorem 1). Plugging in the participation constraints, we get the following maximization program:

$$\max_{a_1,a_2} \quad (1+\delta)a_1 + a_2 - \frac{a_1^2}{2} - \frac{a_2^2}{2} \tag{6}$$

This program leads to the optimal activity choices: $a_1^{\mathrm{S}} = (1+\delta)$ and $a_2^{\mathrm{S}} = 1$. The respective surplus is:

$$\pi^{\mathrm{S}} := \frac{(1+\delta)^2}{2} + \frac{1}{2}.$$

The analysis implies that task separation is optimal if and only if

$$\pi^{\mathrm{S}} > \pi^{\mathrm{B}} \quad \text{or} \quad \frac{(1+\delta)^2+1}{(2+\delta)^2} > \frac{1}{2-\gamma}.$$

This result reflects the basic trade-off between more control and exploiting gains of complementarity: The left-hand side increases in δ and the right-hand side in γ, which yields the testable prediction that task separation is more likely if control is important (δ is large) and complementarities are small (γ small).

6.4 Implementability in Single-Task Models

In the pioneering works on the moral-hazard model by Holmström (1979, 1982) and Shavell (1979) and many ensuing contributions, the agent faces only one decision: how much effort to exert: $a \in \mathbb{R}$. The focus of this literature is not on the failure to induce agents to engage in desired behavior—for a vivid account of such failures from the same time, see Kerr (1975)—but on the trade-off between incentives and insurance.[22] In fact, restrictions on implementability cannot be meaningfully discussed in most single-task moral-hazard models because they typically assume that observable output stochastically increases in effort. This means that the signal is an injective function

[22] See Prendergast (1999, 2002) for a critical empirical overview on the evidence concerning this trade-off.

of the agent's effort choice, so that any identification problem is due to multitasking (Corollary 4). Since there is by definition only one task, the agent's activity is identified and implementability unrestricted (Theorem 1).[23] The next section presents a single-task model with restricted implementability.

6.5 Identification with Biased Reports

In many circumstances, incentives are based on subjective evaluations, e.g., written reports. The expert writing the report rarely assesses behavior in exactly that way which is required to induce a desired activity choice; signals may be biased. This section shows that a second even more biased report can then be valuable.

In order to support this claim, consider a risk-neutral agent whose activity choice, $a \in \mathbb{R}$, affects the probability of report j being favorable ($S_j = 1$) or not ($S_j = 0$) with $j = 1, 2$. More specifically, assume that the activity is evaluated against a benchmark, α_j, and that the probability of a favorable report, $P(S_j = 1|\mu_j)$, decrease in the distance μ_j between the activity choice, a, and this benchmark: $\mu_j(a) = (a - \alpha_j)^2$. For simplicity, let the principal's benefit from the activity be the activity itself and the agent's cost be quadratic. Then, the joint surplus amounts to: $a - c(a) = a - \frac{a^2}{2}$, which is maximized for $a^* = 1$. An ideal signal would thus reveal by how far the agent misses the first-best choice $a^* = 1$. In order to represent the bias, let the benchmarks be too low: $0 < \alpha_2 < \alpha_1 < 1$. Which report will the principal

[23] The formal proof involves an additional step because the early moral-hazard literature uses the so-called Mirrlees representation of the signal structure, which first has to be transformed into state-space representation—for the details, how this can be achieved, see Conlon (2009).

use in an optimal contract and when is a second report valuable?

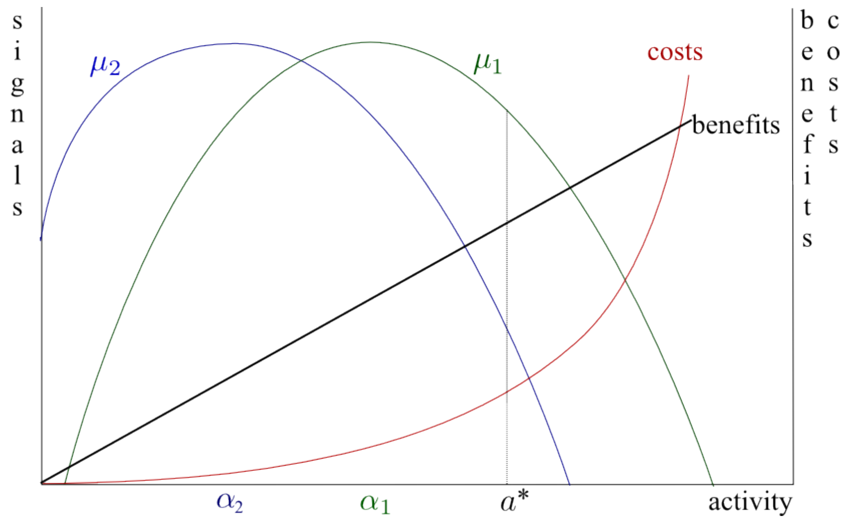

Figure 2: *Using a biased signal with parameter μ_1, the largest activity level that can be induced is α_1. By adding an even more biased signal with parameter μ_2, agents' behavior becomes identified and the first-best effort, a^*, can be induced.*

Each of the two signals alone is not injective in the activity choice—see also Figure 6.5. For example, $\mathring{a} := \alpha_j + 1$ and $\tilde{a} := \alpha_j - 1$ both lead to $\mu_j = 1$. With only one signal, the agent's activity is thus not identified and implementation is limited. Moreover, the problem is not due to multi-tasking and task assignment offers no viable identification strategy. Activity choice $\mathring{a}$ can only be induced if some linear combination of the marginal effect on the signal, λ, equals marginal costs: $\lambda(-2(\mathring{a} - \alpha_j)) = \mathring{a}$ (by Proposition 1). Solving for $\mathring{a}$ yields: $\mathring{a}(\lambda) = \frac{\alpha_j}{\frac{1}{2\lambda}+1}$. While the activity choice increases in λ, it is bounded: $\lim_{\lambda\to\infty} \mathring{a}(\lambda_1) = \alpha_j$. Factoring in the agent's participation constraint, the principal's net gain equals the joint surplus: $a - \frac{a^2}{2}$. This net gain strictly increases in a for $a < \alpha_j < 1$ and thus attains its supremum at $a = \alpha_j$. If the principal had to select one report, she would take the less biased one and obtain a surplus that is arbitrarily close to

$$\pi^{1\mathrm{R}} := \alpha_1 - \frac{\alpha_1^2}{2}.$$

With two reports, the identification problem is solved. In order to see this, consider the marginal effect matrix:

$$D_a\mu = \begin{pmatrix} -2(a-\alpha_1) \\ -2(a-\alpha_2) \end{pmatrix}.$$

This matrix has a minimal rank of one because one of the two entries is always different from zero (since $\alpha_1 \neq \alpha_2$). Hence, a single agent with one task faces one independent signal and his activity is identified (by Proposition 2). Any activity can thus be implemented, in particular, the first-best choice, $a^* = 1$. The surplus with both reports thus amounts to:

$$\pi^{2\mathrm{R}} := \frac{1}{2}.$$

Accordingly, the additional report has a value of at least

$$\pi^{2\mathrm{R}} - \pi^{1\mathrm{R}} = \frac{1}{2} - \alpha_1 + \frac{\alpha_1^2}{2} = \frac{1}{2}(1-\alpha_1)^2.$$

This value is independent of α_2 and strictly positive (as long as the first report is biased). The second report thus adds value irrespective of its bias and this value becomes larger, the larger the bias of the first report, i.e., the smaller α_1.

6.6 Identification in Holmström and Milgrom (1991)

Corts (2007) claims that in large parts of Holmström and Milgrom's seminal paper on multi-tasking (1991) are free from 'multitask problem[s]' once randomness is removed. This section clarifies, supports and extends Corts' claim: Holmström and Milgrom's article features no identification problem caused by multi-tasking. At first glance, this observation seems to contradict that Holmström and Milgrom's article is known for having provided the first formal analysis of multi-tasking problems. The seeming contradiction results from the fact that the term 'multi-tasking problem' refers to two very different phenomena. On the one hand, multi-tasking can negatively affect the trade-off between insurance and incentives, which is the problem examined by Holmström and Milgrom. On the other hand, there is the problem at the heart of the present article: restricted implementability caused by identification problems due to multi-tasking.[24]

The following substantiates the claim that multi-tasking causes no restrictions on implementability in any of the model variations discussed in Holmström and Milgrom's article. These variations employ either what Corts (2007) calls the *one-signal-per-task assumption* or what may be called a *no-signal-for-some-task assumption*.[25] The first assumption describes that for each task i, there is a signal j that strictly increases in the agent's choice at that task and is unaffected by other choices: $\frac{\partial \mu_j}{\partial a_i} > 0$ if $i = j$ and $\frac{\partial \mu_j}{\partial a_i} = 0$ otherwise. This assumption implies that there are as many independent signals as tasks: $\text{rank} D_a \mu = n$. So even if a single agent is responsible for all n tasks, this agent's activity is identified (by Proposition 2) and any activity choice can be implemented (by Theorem 1). The second assumption means

[24] See Inderst and Ottaviani (2009), for a recent contribution using the term 'multitasking problem' in this second sense.

[25] For example, Holmström and Milgrom's job design model uses the one-signal-per-task assumption, while their home contractor model or asset enhancement models employ the no-signal-for-some-task assumption.

that some task i affects no signal j: $\frac{\partial \mu_j}{\partial a_i} = 0$ for all j. This assumption clearly leads to an identification problem. However, this problem cannot be alleviated by eliminating multi-tasking or any other form of task assignment (by Corollary 4).[26]

The models discussed in Holmström and Milgrom (1991) thus concern two extreme situations: those in which *any* and those in which *no* assignment of tasks leads to identification. Examining only these extremes, identification by task assignment cannot be studied. Accordingly, Holmström and Milgrom's results on optimal job design are not driven by the desire to induce new activities.

6.7 Multi-Tasking with Linear Signals

Holmström and Milgrom (1991) inspired a burgeoning literature in accounting (see e.g. Feltham and Xie, 1994; Datar, Cohen Kulp, and Lambert, 2001) as well as contributions to labor economics (Baker, 2000, 2002; Schnedler, 2008), which examine multi-tasking in a particularly tractable framework. Among other things, this framework assumes that signals are linear in activity choices. The literature starts with the observation that the principal's power to implement activities is restricted and proceeds to examine how congruency between signals and the principal's benefit affect optimal (linear) contracts. It neither identifies the source of restricted implementability nor does it consider whether and when task assignment may solve the problem.

[26] Incidentally, the legendary 'multi-tasking' examples mentioned in the introduction also feature activity dimension that do not affect observable results (no-signal-for-some-task-assumption); these famous implementability problems are thus also not caused by multi-tasking.

With the tools introduced here, this gap can be closed. Given linear signals, task assignment can almost always solve identification problems. The reason is the following. Since signals are linear in activities, $\mu_j(a) = \mu_{j1}a_1 + \ldots + \mu_{jn}a_n$, signal j is either independent ($\mu_{ji} = 0$) or injective ($\mu_{ji} \neq 0$) in task i. The former almost never happens, i.e., the event has Lebesgue-measure zero. As soon as some signal j is affected by task i, however, the whole parameter vector, μ, is task-wise injective. Any identification problems are thus due to multi-tasking and can be solved by task assignment (by Corollaries 3 and 4). In other words, the implementation problems in most of the later multi-tasking models are indeed caused by multi-tasking.

7 Conclusion and Discussion

In the first step of their famous analysis of the principal agent-problem, Grossman and Hart (1983) determine the least-costly way of implementing a specific activity. They also observe that it may sometimes be prohibitively costly to implement an activity. However, they neither examine when this is the case nor how these costs are affected by task assignment. The present paper addresses these questions. It advocates incorperating the notion of *identification* into agency theory in addition to other originally econometric concepts such as 'likelihood ratio' (Mirrlees, 1979; Rogerson, 1985), 'sufficient statistic' (Holmström, 1979, 1982), and 'hazard rate' (see e.g. Gibbons, 1987). Identification describes the signal imperfection that restricts implementability: any activity choice can be induced if activities are identified (Theorem 1). Task assignment can identify activities and ensure unlimited implementability whenever changes in every dimension of an activity affect observables (Corollary 4). The characteristics of task assignments that solve the identification problems echo Tinbergen's dictum (1952) that there need to be as many instruments as goals: the number of independent signals for an agent must be at least as large as his number of tasks (Proposition 2).

Identification by task assignment is driven by two crucial assumptions. First, it must be possible to enforce decision rights. For example, agents must be prevented from carrying out tasks that they are not supposed to, while agents who should work on these tasks must be able to do so. This points to one possible reason why restricted access to tools and production sites is common in many firms and organizations: it gives the management more control over how outcomes are achieved. Second, if agents collude, then identification problems cannot be solved by partitioning tasks differently. Identification becomes possible because partitions generate a non-cooperative game in which each agent's equilibrium strategy can be separately manipulated. If agents were colluding, which requires that they can (formally or informally) condition rewards on activities although the mechanism designer cannot, they would jointly agree to chose the least-costly activity in order to produce a given outcome; no other way of producing this outcome could then be implemented.

The results from the present paper uncover why certain task assignments, which are proposed in the literature, such as advocates, specialists, or joint accountability, are advantageous. The reasons are not task conflicts or accountability issues; the suggested task assignments overcome identification problems. From the results, it follows that separating tasks in order to identify agents' activities may reduce agency costs (even if the signal structure remains the same).[27] Using this insight and complementing existing models, optimal task bundling can be regarded as the result of a trade-off between more control, on the one hand, and exploiting complementarities, on the other hand. Although implementability is unlimited in most traditional single-task models, it became apparent that single-task models may well exhibit identification problems, in which case biased signals can be beneficial. In addition, results help to pin down a specific type of problem caused by multi-tasking. Multi-tasking can create identification problems and may thus limit implementability. This type of multi-tasking problem is latent in many models but very different from that described by Holmström and Milgrom (1991). As shown, multi-tasking causes no identification problems in their article and hence does not restrict which activities can be implemented.

The present paper shows that given identification, rewards can be set such that any desired activity is induced as a Nash equilibrium. This leaves several questions open. Can unique implementation be achieved? Can activities be implemented in dominant strategies or as an attractor of best-response dynamics? All these questions are beyond the scope of the paper and left for future research. While the paper links implementability, identification, and task assignment in a rather general framework, it only provides a first-step in the analysis of implementability in moral hazard models. The generality of the structure only permits relatively fundamental statements about existence and form of task assignments that solve identification problems. Finer predictions require additional assumptions, for example, on which tasks are

[27]Dealing with performance measures in accounting, Hemmer (1998) suggests that the informational content of the same performance measure system can be altered when the organizational design changes.

separable and how activities are reflected in signals.

References

Alonso, R., and N. Matouschek (2008): "Optimal Delegation," *Review of Economic Studies*, 75(1), 259–293.

Amershi, A. H., and J. S. Hughes (1989): "Multiple Signals, Statistical Sufficiency, and Pareto Orderings of Best Agency Contracts," *RAND Journal of Economics*, 20(1), 102–112.

Baker, G. (2000): "The Use of Performance Measures in Incentive Contracting," *American Economic Review*, 90(2), 415–420.

——— (2002): "Distortion and Risk in Optimal Incentive Contracts," *Journal of Human Resources*, 37(4), 728–751, Special Issue on Designing Incentives to Promote Human Capital.

Battaglini, M. (2006): "Joint production in teams," *Journal of Economic Theory*, 130(1), 138 – 167.

Bolton, P., and M. Dewatripont (2005): *Contract theory*. MIT Press.

Borland, J., and J. Eichberger (1998): "Organizational Form Outside the Principal-Agent Paradigm," *Bulletin of Economic Research*, 50(3), 201–27.

Conlon, J. R. (2009): "Two New Conditions Supporting the First-Order Approach to Multisignal Principal-Agent Problems," *Econometrica*, 77(1), 249–278.

Corts, K. S. (2007): "Teams versus Individual Accountability: Solving Multitask Problems Through Job Design," *RAND Journal of Economics*, 38(2), 467–479.

Datar, S., S. Cohen Kulp, and R. A. Lambert (2001): "Balancing Performance Measures," *Journal of Accounting Research*, 39(1), 75–92.

Dewatripont, M., I. Jewitt, and J. Tirole (2000): "Multitask Agency Problems - Focus and Task Clustering," *European Economic Review*, 44(4-6), 869–877.

Dewatripont, M., and J. Tirole (1999): "Advocates," *Journal of Political Economy*, 107(1), 1–39.

Feltham, G. A., and J. Xie (1994): "Performance Measure Congruity and Diversity in Multi-Task Principal/Agent Relations," *The Accounting Review*, 69(3), 429–453.

Fudenberg, D., D. Levine, and E. Maskin (1994): "The Folk Theorem with Imperfect Public Information," *Econometrica*, 62(5), pp. 997–1039.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. The MIT Press, Cambridge, Massachusetts.

Gibbons, R. (1987): "Piece-Rate Incentive Schemes," *Journal of Labor Economics*, 5(4), 413–429.

Grossman, S. J., and O. D. Hart (1983): "An Analysis of the Principal-Agent Problem," *Econometrica*, 51(1), 7–46.

Hemmer, T. (1998): "Performance measurement systems, incentives, and the optimal allocation of responsibilities," *Journal of Accounting and Economics*, 25(3), 321–347.

Hermalin, B., and M. Katz (1991): "Moral Hazard and Verifiability: The Effects of Renegotitation in Agency," *Econometrica*, 59(6), 1735–1753.

Holmström, B. (1979): "Moral Hazard and Observability," *Bell Journal of Economics*, 10(1), 74–91.

——— (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13(2), 324–340.

Holmström, B., and P. Milgrom (1991): "Multitask Principal-Agent-Analysis: Incentive Contracts, Asset Ownership, and Job Design," *Journal of Law, Economics, and Organization*, 7, 24–52, special issue.

Inderst, R., and M. Ottaviani (2009): "Misselling Through Agents," *American Economic Review*, 99, 883–908.

Kerr, S. (1975): "On the Folly of Rewarding A While Hoping for B," *Academy of Management Journal*, 18(4), 769–783.

Kragl, J., and A. Schöttner (2011): "Wage Floors and Optimal Job Design," Discussion Paper 1746127, SSRN eLibrary.

Legros, P., and H. Matsushima (1991): "Efficiency in partnerships," *Journal of Economic Theory*, 55(2), 296–322.

Macho-Stadler, I., and D. Perez-Castrillo (1997): *An Introduction to the Economics of Information: Incentives and Contracts*. Oxford University Press, Oxford.

Mirrlees, J. A. (1974): "Notes on Welfare Economics, Information and Uncertainty," in *Essays on Economic Behaviour Under Uncertainty*, ed. by M. Balch, D. McFadden, and S. Wu, pp. 243–258. Amsterdam.

——— (1979): "The Implications of Moral Hazard for Optimal Insurance," Seminar given at converence held in honor of Karl Borch, Bergen, Norway.

——— (1999): "The Theory of Moral Hazard and Unobservable Behavior (Part 1)," *Review of Economic Studies*, 66, 3–21, reprint of a manuscript from 1975.

Mookherjee, D. (1997): "Wealth Effects, Incentives, and Productivity," *Review of Development Economics*, 1(1), 116–33.

——— (2006): "Decentralization, Hierarchies, and Incentives: A Mechanism Design Perspective," *Journal of Economic Literature*, 44(2), 367–390.

Prendergast, C. (1999): "The Provision of Incentives in Firms," *Journal of Economic Literature*, XXXII, 7–63.

——— (2002): "The Teneous Trade-Off Between Risk and Incentives," *Journal of Political Economy*, 110(5), 1071–1102.

Raith, M. (2008): "Specific Knowledge and Performance Measurement," *The RAND Journal of Economics*, 39(4), 1059–1079.

Ratto, M., and W. Schnedler (2008): "Too Few Cooks Spoil the Broth: Division of Labour and Directed Production," *The B.E. Journal of Economic Analysis & Policy*, 8(1, Topics), Article 27.

Rogerson, W. P. (1985): "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53(6), 1357–1367.

Ross, S. A. (1973): "The Economic Theory of Agency: The Principal's Problem," *American Economic Review*, 63(2), 134–39.

Schnedler, W. (2008): "When is It Foolish to Reward for A While Benefiting from B?," *Journal of Labor Economics*, 26(4), 595–619.

——— (2010): "Hidden Action, Identification, and Organization Design," Discussion Paper 505, Department of Economics, University of Heidelberg.

——— (2011): "You Don't Always Get What You Pay For," *German Economic Review*, 12(1), 1–10.

Shavell, S. (1979): "Risk-Sharing and Incentives in the Principal-Agent Relationship," *Bell Journal of Economics*, 10(1), 55–73.

Spence, M., and R. Zeckhauser (1971): "Insurance, Information, and Individual Action," *American Economic Review*, 61(2), 380–87.

Strausz, R. (1999): "Efficiency in Sequential Partnerships," *Journal of Economic Theory*, 85(1), 140 – 156.

Thiele, H., and A. Wambach (1999): "Wealth Effects in the Principal Agent Model," *Journal of Economic Theory*, 89(2), 247–260.

Tinbergen, J. (1952): *On the Theory of Economic Policy*. North-Holland Publishing Company, Amsterdam.

A Proofs

The proofs are listed in the order of appearance of the respective results.

Proof Proposition 1. The proof relies on various auxiliary results that are listed in Appendix B. Fix a partition $\mathcal{P}$ and a signal structure $\mathcal{S}$. Suppose all agents engage in the activity choice to be implemented, $\mathring{a}$. The proof then has to show that an arbitrary agent, say agent l, has no incentive to deviate if and only if conditions (3) are met. It uses marginal arguments, where Lemma 1 ensures that agents' expected utility is indeed differentiable in their activity (even if rewards are not). It is relatively straightforward to show that conditions (3) are necessary to prevent agent l from deviating. Any (interior) choice by the agent must meet his first-order conditions, which can be rewritten using the separability in signals and utility in form of conditions (3)—the details can be found in Lemma 2 in Appendix B. Showing that the conditions are also sufficient involves two steps. First, it is proven that for an appropriate choice of linear rewards r^l, the first-order conditions can be met. Second, we derive that the agent's expected utility is concave given any linear scheme, so that the first-order conditions are also sufficient.

Let us begin by showing that once conditions (3) hold, it is possible to find linear rewards $r^l : \mathbb{R}^k \to \mathbb{R}$, where $r^l(s) = r^l_0 + r^l_1 s_1 + \dots r^l_k s_k$ for all l such that the first-order conditions are met. In order to avoid wealth effects, define r^l_0 as an implicit function of $(r^l_1, \dots, r^l_k)$ such that the outside option is met for agent l at $\mathring{a}$:

$$\mathrm{E}_\epsilon \left[u^l(r^l_0 + r^l_1 S_1 + \dots + r^l_k S_k, c^l(a)) \right] \Big|_{a=\mathring{a}} = \underline{u}^l. \tag{7}$$

Since condition (3) holds for $\mathring{a}$, there is some λ^l such that

$$\lambda^l D_{a^l}\mu(a)|_{a=\mathring{a}} = D_{a^l}c^l(a)|_{a=\mathring{a}}. \tag{8}$$

Suppose that the following were true for some $(r_1^l, r_2^l, \ldots, r_k^l)$:

$$\begin{aligned} &\frac{d}{d\mu_j}\mathrm{E}_\epsilon\left[u^l(r_0^l + r_1^l S_1(\mu_1(a),\epsilon_1) + \ldots + r_k^l S_k(\mu_k(a),\epsilon_k), c^l)\right]\Big|_{a=\mathring{a}} \\ &= \lambda_j^l \cdot \mathrm{E}_\epsilon\left[-\frac{\partial}{\partial c^l}u^l(r_0^l + r_1^l S_1(\mu_1(a),\epsilon_1) + \ldots + r_k^l S_k(\mu_k(a),\epsilon_k), c^l(a))\right]\Big|_{a=\mathring{a}}. \end{aligned} \tag{9}$$

Then, λ^l can be replaced in equation (8), so that the first-order conditions (2) are met at $a = \mathring{a}$:

$$\begin{aligned} & D_\mu \mathrm{E}_\epsilon\left[u^l(r^l(S(\mu,\epsilon)), c^l)\right] \cdot D_{a^l}\mu(a)\big|_{a=\mathring{a}} \\ = \quad & -\frac{d}{dc^l}\mathrm{E}_\epsilon\left[u^l(r^l(S(\mu,\epsilon)), c^l)\right] \cdot D_{a^l}c^l(a)\Big|_{a=\mathring{a}}, \text{ for all } l. \end{aligned} \tag{2}$$

All that remains to be shown is that $(r_1^l, \ldots, r_k^l)$ can indeed be chosen in dependence of λ_j^l such that for all signals j equation (9) holds. In order to do so, we exploit that both sides of equation (9) are continuous in r_j^l, that the left-hand side is bounded away from zero by the linear function $r_j^l \cdot \kappa_j^l$, with some positive κ_j^l (see Lemma 3 in Appendix B), while the absolute value of the right-hand side is smaller than $|\lambda_j^l|$ because $0 > \frac{\partial u}{\partial c^l} \geq -1$. Taken together, there is some $(\mathring{r}_1^l, \ldots, \mathring{r}_k^l)$ such that the both sides in equation (9) are equal and the first-order conditions hold—for an illustration with positive λ_j^l see Figure 3.

In the second and last step, it will be shown that first-order conditions are also sufficient because agent l's expected utility is concave in a^l given linear rewards. The reason is the following. By definition $S(\mu(a), e)$ is concave

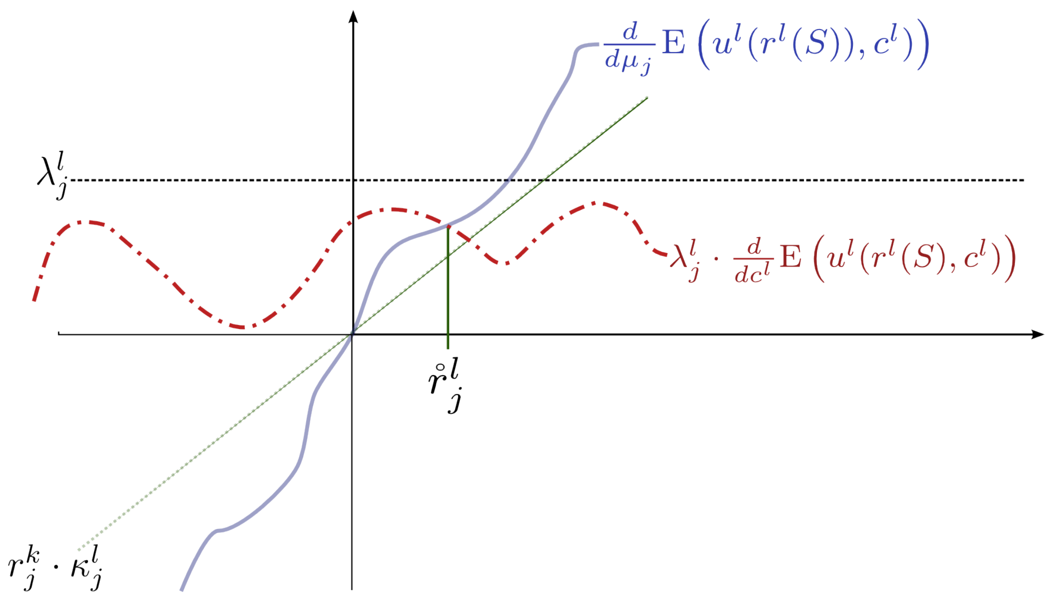

Figure 3: Existence of a reward rate, $\mathring{r}_j^l$, such that agent l's first-order condition are met.

in a for almost all realizations e of ϵ. Hence, $r^l(S(\mu(a), e)) = r_0^l + r_1^l S_1 + \ldots + r_k^l S_k$ is concave in a for almost all e. Moreover, $c^l(a)$ is convex in a and $u^l(r^l, c^l)$ is concave in r^l and c^l by definition. Since $u^l(r^l, c^l)$ decreases in c, $u^l(r^l(S(\mu(a), e)), c^l(a)))$ is concave in a for almost all e. The concavity is maintained when integrating over ϵ, so that $\mathrm{E}_\epsilon\left[u^l(r^l(S(\mu(a), \epsilon), c^l(a)))\right]$ is concave in a and consequently also in any sub-vector a^l. □

Proof Proposition 2. First, we show that rank $(D_{a^l}\mu(a)) < |N^l|$ for some l and a implies an identification problem. Let a^l denote the respective activity by agent l. Due to rank$(D_{a^l}\mu(a)) < |N^l|$, the set $\{\hat{a}^l | D_{a^l}\mu(a)\hat{a}^l = (0, \ldots, 0)'\}$ is not empty. Since A is an open set, the non-emptiness implies that there is a value $\tilde{a}^l$ (near a^l) with $\tilde{a}^l \neq a^l$ but $\mu(\tilde{a}^l, a) = \mu(a^l, a^{-l})$ and that the mechanism designer faces an identification problem.

Second, we prove that an identification problem implies $\text{rank}(D_{a^l}\mu(a)) < |N^l|$ for some l and a. By definition, an identification problem is present if there are differing activities a^l and $\tilde{a}^l$ for some agent l such that $\mu(a^l, a^{-l}) = \mu(\tilde{a}^l, a^{-l})$. Since μ is continuously differentiable and A is convex, there is an activity $\hat{a}^l$ such that $0 = \mu(a^l, a^{-l}) - \mu(\tilde{a}^l, a^{-l}) = D_{a^l}\mu(\hat{a})(a^l - \tilde{a}^l)$. This, however, implies that $\text{rank}\left(D_{a^l}\mu(\hat{a})\right) < |N^l|$. □

Proof Theorem 1. First, we prove that an identification problem leads to limited implementability. Observe that strict convexity of costs $c^l(a)$ and concavity of $\mu(a)$ implies that there is a *unique* least costly way for agent l to produce $\bar{\mu}$:

$$\arg \min_{\hat{a}^l \in \{\check{a}^l | \mu(\check{a}^l, a^{-l}) = \bar{\mu}\}} c^l(\hat{a})|_{\hat{a}=(\hat{a}^l, a^{-l})}.$$

Suppose there is an identification problem, then there is some $\bar{\mu}, \tilde{a}^l$ and a^l such that $\bar{\mu} = \mu(a^l, a^{-l}) = \mu(\tilde{a}^l, a^{-l})$ but $\tilde{a}^l \neq a^l$. Since the least costly way to produce $\tilde{\mu}$ is unique, either a^l or $\tilde{a}^l$ cannot be induced and implementability is limited.

Second, we show that any activity can be implemented in the absence of identification problems. If there is no identification problem, $\text{rank}\left(D_{a^l}\mu(a)\right) \geq |N^l|$ for all l and a by Proposition 2. Take an arbitrary activity a and agent l and let a^l be agent l's decision given a^{-l}. By definition, the rank of the matrix cannot be larger than the number of rows, k, so that $k \geq \text{rank}(D_{a^l}\mu) \geq |N^l|$, which directly implies that $min(k+1, |N^l|) = |N^l|$. Accordingly, the rank of the following matrix with $k+1$ rows and $|N^l|$ colums can at most be $|N^l|$:

$$|N^l| \geq \text{rank}\begin{pmatrix} D_{a^l}\mu(a) \\ D_{a^l}c^l(a) \end{pmatrix}. \tag{10}$$

On the other hand, this rank cannot be smaller than that of the first k rows:

$$\text{rank}\begin{pmatrix} D_{a^l}\mu(a) \\ D_{a^l}c^l(a) \end{pmatrix} \geq \text{rank}(D_{a^l}\mu(a)). \tag{11}$$

Together, (10) and (11) imply:

$$|N^l| \geq \text{rank}\begin{pmatrix} D_{a^l}\mu(a) \\ D_{a^l}c^l(a) \end{pmatrix} \geq \text{rank}(D_{a^l}\mu(a)) \geq |N^l|.$$

But this is only possible if

$$\text{rank}\begin{pmatrix} D_{a^l}\mu(a) \\ D_{a^l}c^l(a) \end{pmatrix} = \text{rank}(D_{a^l}\mu(a)).$$

Applying Corollary 2 then yields that a^l can be implemented with agent l. □

Proof Corollary 3. By definition, identification problems due to multi-tasking can be solved by the finest partition of tasks $\{\{1\},\ldots,\{n\}\}$. Thus, there exists a task assignment that solves the identification problem. Conversely, if some task assignment solves the identification problem, there is a partition $\mathcal{P}$ with enough independent signals for each agent l: $\text{rank}(D_{a^l}\mu(a)) \geq |N^l|$ by Proposition 2. Consider partitioning agent l's $|N^l|$ tasks amongst $|N^l|$ different agents. Then, for each of these agents $\tilde{l} \in N^l$, $D_{a^{\tilde{l}}}\mu(a)$ is a $(k,1)-$vector and $\text{rank}(D_{a^l}\mu(a)) \geq |N^l|$ implies $\text{rank}(D_{a^{\tilde{l}}}\mu(a)) = 1$. Since each of the agents only carries out one task $|N^{\tilde{l}}| = 1$ and $\text{rank}(D_{a^{\tilde{l}}}\mu(a)) = 1 = |N^{\tilde{l}}|$, Proposition 2 implies that there is no identification problem. The identification problem thus disappears with the finest partition and is hence caused by multi-tasking.

□

Proof Corollary 4. Suppose there is an identification problem given partition $\mathcal{P}$. Eliminate multi-tasking by considering the finest partition $\{\{1\},\ldots,\{n\}\}$. The finest partition solves the identification problem if and only if for all l and $\tilde{a}^l \neq a^l$ it follows that $\mu(a^l, a^{-l}) \neq \mu(\tilde{a}^l, a^{-l})$. Since each agent l carries out exactly one task i, this is true if and only if $\mu(\ldots, a_i, \ldots) \neq \mu(\ldots, \tilde{a}_i, \ldots)$ for all i and $a_i \neq \tilde{a}_i$. □

Proof Proposition 3. The proof works by contradiction. Let $\tilde{a} \in \arg\min_a \operatorname{rank}(D_a\mu(a))$. Suppose $\tilde{k}m < n$, where $\tilde{k} := \min_a \operatorname{rank}(D_a\mu(a))$. Then,

$$\underbrace{\text{rank } D_a\mu(a)|_{a=\tilde{a}} + \ldots + \text{ rank} D_a\mu(a)|_{a=\tilde{a}}}_{m \text{ summands}} < n = \sum_{l=1}^{m} |N^l|.$$

This implies:

$$\sum_{l=1}^{m} \text{ rank } D_{a^l}\mu(a)|_{a=\tilde{a}} < \sum_{l=1}^{m} |N^l|. \tag{12}$$

Now, take an arbitrary partition $\mathcal{P}$ and suppose that there is no identification problem. By Proposition 2, it must then hold that rank $(D_{a^l}\mu(a)) \geq |N^l|$ for all l. This, however, contradicts equation (12). Consequently, the identification problem persists for any partition. □

Proof Proposition 4. Since $\tilde{k} := \min_a \operatorname{rank}(D_a\mu)$, there are at least $\tilde{k}$ independent columns of $D_a\mu$ for all a. Assign the tasks belonging to these columns to the first agent. For this agent, it now holds that rank $(D_{a^1}\mu(a)) \geq \tilde{k} = |N^1|$ for all a and his activities are identified. Since $m > n - \tilde{k}$, there remain at least $n - \tilde{k}$ agents who are not yet assigned to tasks. Assign each of these agents to one of the remaining $n - \tilde{k}$ tasks. Since the identification problem is caused by multi-tasking, the activities of these agents are also identified. □

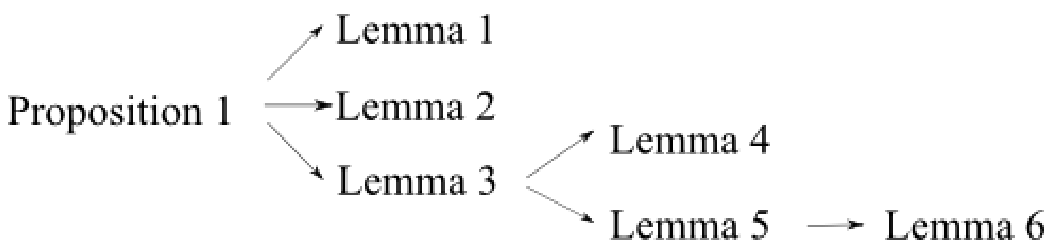

Figure 4: Dependencies of Auxiliary Results: Proposition 1 relies on Lemma 1 to 3, Lemma 3 depends on Lemma 4 and 5, and Lemma 5 on Lemma 6

B Auxiliary Results for Proposition 1

This section presents various lemmas that lay the foundations for Proposition 1. Lemma 1 ensures that agent l's expected utility is differentiable in his activity even if the rewards r^l are not differentiable in signals. Lemma 2 guarantees that the central condition in Proposition 1, conditions (3), are necessary. Lemma 3 is needed for the proof that the conditions are also sufficient; it binds the marginal effect of the parameter vector on agent's expected utility away from zero. Lemma 3 is based on Lemma 4 and 5, where the latter relies on Lemma 6—see Figure 4.

Lemma 1 (Differentiability of Expected Utility)**.** *The expected utility is differentiable in the activity vector.*

Proof. Let $\mathcal{S}^{\mathrm{AC}}$ be the set of signal realizations at which S is absolutely continuous and $\mathcal{S}^{\mathrm{D}}$ be the set of discrete signal realizations. Then, the expected utility can be written as

$$\mathrm{E}\left[u^l(r^l(S),c)\right] = \int_{s\in\mathcal{S}^{\mathrm{AC}}} u^l(r^l(s),c)f(s,\mu)ds \tag{13}$$

$$+\sum_{s\in\mathcal{S}^{\mathrm{D}}} u^l(r^l(s),c)P(S=s,\mu), \tag{14}$$

where f is the joint density of S, $P(S=s,\mu)$ the joint probability distribution given μ and c are the agent's costs. Using that the support is constant in μ, the derivative of the expected utility in μ can be found by directly taking derivatives of the arguments of the integral and sum, which yields:

$$\frac{d}{d\mu}\mathrm{E}\left[u^l(r^l(S),c)\right] = \int_{s\in\mathcal{S}^{\mathrm{AC}}} u^l(r^l(s),c)\frac{d}{d\mu}f(s,\mu) \tag{15}$$

$$+\sum_{s\in\mathcal{S}^{\mathrm{D}}} u^l(r^l(s),c)\frac{d}{d\mu}P(S=s,\mu). \tag{16}$$

Since μ and c are themselves differentiable in a, the derivative of the expected utility in the activity vector can be obtained using the chain rule.

□

Lemma 2 (Necessary condition for implementation)**.** *Given signal structure $\mathcal{S}$ and partition $\mathcal{P}$, an activity $\mathring{a}$ is only implementable if for all agents some linear combination of the activity's marginal effect on the parameter equals that on costs:*

$$\mathring{a}\in\mathcal{I}\Rightarrow\forall l\exists\lambda^l\in\mathbb{R}^k:\ \textit{such that}\ \lambda^l\ D_{a^l}\mu(a)\big|_{a=\mathring{a}} = D_{a^l}c^l(a)\Big|_{a=\mathring{a}}. \tag{17}$$

Proof. Since we are looking for an inner maximizer, the first-order conditions are necessary. Using the separability assumption, these conditions are equivalent to (2). Next suppose that the condition on the right-hand side of (17) is violated and show that first-order conditions cannot be met then. Suppose for some agent l and all $\lambda^l : \lambda^l D_{a^l}\mu(a)|_{a=\mathring{a}} \neq D_{a^l}c^l(a)|_{a=\mathring{a}}$. Now choose $\tilde{\lambda}^l := -\frac{D_\mu \mathrm{E}_\epsilon\left[u^l(r^l(S(\mu,\epsilon)),c^l)\right]}{\frac{d}{dc^l}\mathrm{E}_\epsilon\left[u^l(r^l(S(\mu,\epsilon)),c^l)\right]}\Bigg|_{a=\mathring{a}}$, where the denominator is strictly negative because of $\frac{\partial u^l}{\partial c^l}<0$. Then,

$$\tilde{\lambda}^l D_{a^l}\mu(a)\neq D_{a^l}c^l(a),$$

which implies directly that the first-order condition are violated, so that $\mathring{a} \notin \mathcal{I}$. ☐

Lemma 3. *Fix μ, and c, and and consider linear rewards $r^l(\cdot)$ such that the agents' outside options are met. Then, there is some $\kappa_j^l \in \mathbb{R}$ with $\kappa_j^l > 0$ such that:*

$$\left| \frac{d}{d\mu_j} E_S \left[u^l(r^l(S), c) \right] \right|_{a=\mathring{a}} \geq |r_j^l| \cdot \kappa_j^l. \tag{18}$$

Proof. Define $S_{-j} := (S_1, \ldots, S_{j-1}, S_{j+1}, \ldots, S_k)$ and let F_{-j} be the respective cumulative distribution. Denote the c.d.f. of S_j given S_{-j} by $F_j(s_j|\mu_j)$ and the respective density by $f_j(s_j|\mu)$. Let us focus on a continuously distributed $S_j(\mu_j, \epsilon_j)$—the analysis in the case of a discrete distribution is analogous. For given costs c, the gain in expected utility from a change from μ_j to some $\tilde{\mu}_j$ with $\tilde{\mu}_j > \mu_j$ can be written as:

$$\begin{aligned}
& \mathrm{E}_S \left[u^l(r^l(S), c) | (\mu_1, \ldots, \mu_{j-1}, \tilde{\mu}_j, \mu_{j+1}, \ldots, \mu_k) \right] - \mathrm{E}_S \left[u^l(r^l(S), c) | \mu \right] \\
&= \iint u^l(r_0^l + \ldots + r_j^l s_j + \ldots + r_k^l s_k, c) \cdot [f_j(s_j|\tilde{\mu}_j) - f_j(s_j|\mu_j)] \, ds_j \, dF_{-j} \\
&= [u^l(r^l(s_j), c)(F_j(s_j|\tilde{\mu}_j) - F_j(s_j|\mu_j))]_{\underline{s}_j}^{\overline{s}_j} \qquad (19) \\
&\quad - \iint \frac{\partial}{\partial r^l} u^l(r^l(s), c) \cdot r_j \cdot [F_j(s_j|\tilde{\mu}_j) - F_j(s_j|\mu_j)] \, ds_j \, dF_{-j},
\end{aligned}$$

where $\underline{s}_j$ and $\overline{s}_j$ are the bounds of the support of S_j and possibly $-\infty$ and $+\infty$. The last equality follows from integration by parts. Moreover the term in (19) is zero because both c.d.f.'s are zero evaluated at the lower bound and one at the upper bound. Dividing both sides by $\tilde{\mu}_j - \mu_j$ and letting $\tilde{\mu}_j \to \mu_j$, we obtain the derivative of the agent's expected utility with respect to μ_j for a given c:

$$
\begin{aligned}
&\frac{d}{d\mu_j}\mathrm{E}_S\left[u^l(r_0^l + r_1^l S_1 + \ldots + r_j^l S_j + \ldots + r_k^l S_k, c)\right] \\
&= -\iint \frac{\partial}{\partial r^l} u^l(r^l(s), c) \cdot r_j^l \cdot \frac{\partial}{\partial \mu_j} F_j(s_j|\mu_j)\, ds_j\, dF_{-j} \\
&= -\iint \frac{\partial}{\partial r^l} u^l(r^l(s), c) \cdot r_j \cdot f_j(s_j|\mu_j) \cdot \frac{\frac{\partial}{\partial \mu_j} F_j(s_j|\mu_j)}{f_j(s_j|\mu_j)}\, ds_j\, dF_{-j} \\
&= r_j^l \cdot \mathrm{E}_S\left[\frac{\partial}{\partial r^l} u^l(r^l(S), c) \cdot \frac{-\frac{\partial}{\partial \mu_j} F_j(S_j|\mu_j)}{f_j(S_j|\mu_j)}\right] \\
&= r_j^l \cdot \mathrm{E}_S\left[\frac{\partial}{\partial r^l} u^l(r^l(S), c)\right] \cdot \gamma_j^l \text{ with } \gamma_j^l > 0, \qquad (20)
\end{aligned}
$$

where the last equality follows from Lemma 4. Given that the agent l's outside option is met, Lemma 5 implies:

$$
\mathrm{E}_S\left[\frac{\partial}{\partial r^l} u^l(r^l(S), c)\right] \geq \delta^l > 0
$$

Using this in (20) yields:

$$
\left|\frac{d}{d\mu_j}\mathrm{E}_\epsilon\left[u^l(r^l(S), c)\right]\right|_{a=\mathring{a}} \geq |r_j^l| \cdot \kappa_j^l \text{ with } \kappa_j^l := \delta^l \cdot \gamma_j^l > 0.
$$

□

Lemma 4. *Given μ, and c, there is a strictly positive γ_j for all rewards $r^l(\cdot)$ such that:*

$$
E_S\left[\frac{\partial}{\partial r^l} u^l(r^l(S), c) \cdot \frac{-\frac{\partial}{\partial \mu_j} F_j(S_j|\mu_j)}{f_j(S_j|\mu_j)}\right] = E_S\left[\frac{\partial}{\partial r^l} u^l(r^l(S), c)\right] \cdot \gamma_j^l,
$$

where $F_j(s_j|\mu_j) = Prob(S_j \leq s_j | S_{j'} = s_{j'}$ for all $j' \neq j)$ and $f_j(s_j) = \frac{\partial}{\partial s_j} F_j(s_j)$.

Proof. Define the real-valued random variable $B_j := \frac{-\frac{\partial}{\partial \mu_j} F_j(S_j|\mu_j)}{f_j(S_j|\mu_j)}$, which only takes on positive values because $\frac{\partial}{\partial \mu_j} F_j(S_j|\mu_j) \geq 0$ (since signal j stochastically increases in its parameter) and $f_j > 0$ on the support of S_j. Using this notation, we get:

$$\begin{aligned}
&\mathrm{E}_S\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\cdot\frac{-\frac{\partial}{\partial\mu_j}F_j(S_j|\mu_j)}{f_j(S_j|\mu_j)}\right]\\
=&\mathrm{E}\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\cdot B_j\,\middle|B_j\geq\beta_j\right]\cdot Prob(B_j\geq\beta_j)\\
&+\mathrm{E}_S\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\cdot B_j\,\middle|B_j<\beta_j\right](1-Prob(B_j\geq\beta_j))\\
\geq&\mathrm{E}_S\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\cdot \beta_j\,\middle|B_j\geq\beta_j\right]\cdot Prob(B_j\geq\beta_j)\\
&+\mathrm{E}_S\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\cdot 0\quad\middle|B_j<\beta_j\right](1-Prob(B_j\geq\beta_j))\\
=&\mathrm{E}_S\left[\frac{\partial}{\partial r^l}u^l(r^l(S),c)\right]\cdot\gamma_j,
\end{aligned}\tag{21}$$

where the inequality follows from $B_j \geq 0$ and the existence of $\gamma_j \in [0, \beta_j]$ in the last line follows from the intermediate value theorem. Moreover, the event $B_j \geq \beta_j$ occurs with positive probability for some $\beta_j > 0$ because S_j is stochastically increasing in μ_j, so that:

$$Prob(B_j > \beta_j) = Prob\left(\frac{-\frac{\partial}{\partial\mu_j}F_j(s_j|\mu_j)}{f_j(s_j|\mu_j)} \geq \beta_j\right) > 0.$$

This, however, means that γ_j has to be strictly larger than zero. □

Lemma 5. *Take a μ, c, and rewards $r^l(\cdot)$ and $\tilde{r}^l(\cdot)$ that yield the same expected utility $\underline{u}^l$:*

$$E\left[u^l(r^l(S(\mu,\epsilon),c))\right] = E\left[u^l(\tilde{r}^l(S(\mu,\epsilon),c))\right] = \underline{u}^l.$$

Then, for some positive δ^l :

$$E\left[\frac{\partial}{\partial r^l}u^l(r^l(S(\mu,\epsilon)),c)\right] \geq \delta^l > 0.$$

Proof. By Lemma 6 there exists a reward for which the cumulative distribution functions of the reward distributions cross:

$$\hat{r}^{\min} := \min\{\hat{r} | Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}) = Prob(\tilde{r}^l(S(\mu,\epsilon)) \leq \hat{r}) > 0\} \subseteq [\underline{r}, \overline{r}].$$

Using this crossing reward $\hat{r}$, we can separate the support:

$$\begin{aligned}
& \mathrm{E}_\epsilon\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c)\right] \\
= \; & \mathrm{E}\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c) \middle| r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}\right] \cdot Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}) \\
& + \mathrm{E}\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c) \middle| r^l(S(\mu,\epsilon)) > \hat{r}^{\min}\right] \cdot Prob(r^l(S(\mu,\epsilon)) > \hat{r}^{\min}) \\
\geq \; & \mathrm{E}\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c) \middle| r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}\right] \cdot Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}),
\end{aligned}$$

where $Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}) > 0$ by the definition of $\hat{r}^{\min}$. Since u^l is concave, $\frac{\partial}{\partial r^l} u^l(r^l, c)$ weakly falls in r^l so that

$$\mathrm{E}\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c) \middle| r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min}\right] \geq \left.\frac{\partial}{\partial r^l} u^l(r^l,c)\right|_{r^l=\hat{r}^{\min}},$$

which is strictly larger than zero because $\frac{\partial}{\partial r^l} u^l(r^l, c) > 0$. Taken together, this implies:

$$\mathrm{E}_\epsilon\left[\frac{\partial}{\partial r^l} u^l(r^l(S(\mu,\epsilon)),c)\right] \geq \underbrace{\left.\frac{\partial}{\partial r^l} u^l(w,c)\right|_{w=\hat{r}^{\min}} \cdot Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}^{\min})}_{=:\delta^l} > 0.$$

□

Lemma 6. *Take a μ, and c and consider rewards $r^l(\cdot)$ and $\tilde{r}^l(\cdot)$ such that:*

$$E\left[u(r^l(S(\mu,\epsilon),c))\right] = E\left[u(\tilde{r}^l(S(\mu,\epsilon),c))\right].$$

Then, there are finite numbers $\underline{r} < \overline{r}$ in the support such that for all $r^l(\cdot)$ and $\tilde{r}^l(\cdot)$, there is a $\hat{r} \in [\underline{r}, \overline{r}]$ in the support of of $r^l(S(\mu,\epsilon))$ and $\tilde{r}^l(S(\mu,\epsilon))$ for which the cumulative distribution function of rewards cross:

$$Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}) = Prob(\tilde{r}^l(S(\mu,\epsilon))) \leq \hat{r}).$$

Proof. The proof works by contradiction. Suppose for all $\overline{r}$, there exist $r^l(\cdot)$ and $\tilde{r}^l(\cdot)$, such that for all $\hat{r} \leq \overline{r}$ it holds that

$$Prob(r^l(S(\mu,\epsilon)) \leq \hat{r}) < Prob(\tilde{r}^l(S(\mu,\epsilon)) \leq \hat{r}),$$

then $r^l(S(\mu,\epsilon))$ first-order stochastically dominates $\tilde{r}^l(S(\mu,\epsilon))$ for values below $\overline{r}$. Together with the assumption that u^l increases in w, this yields:

$$\mathrm{E}\left[u^l(r^l(S(\mu,\epsilon),c)) | r^l(S(\mu,\epsilon)) \leq \overline{r}\right] > \mathrm{E}\left[u^l(\tilde{r}^l(S(\mu,\epsilon),c)) | \tilde{r}^l(S(\mu,\epsilon)) \leq \overline{r}\right].$$

Since this holds for all $\overline{r}$, it also holds if $\overline{r}$ approaches infinity, which means that $Prob(r^l(S(\mu,\epsilon)) \leq \overline{r}) = Prob(\tilde{r}^l(S(\mu,\epsilon)) \leq \overline{r})$ becomes one, so that

$$\begin{aligned}
\mathrm{E}\left[u^l(r^l(S(\mu,\epsilon),c))\right] &= \mathrm{E}\left[u^l(r^l(S(\mu,\epsilon),c)) | r^l(S(\mu,\epsilon)) \leq \overline{r}\right] \\
&> \mathrm{E}\left[u(\tilde{r}^l(S(\mu,\epsilon),c)) | \tilde{r}^l(S(\mu,\epsilon)) \leq \overline{r}\right] \\
&= \mathrm{E}\left[u(\tilde{r}^l(S(\mu,\epsilon),c))\right], \qquad (22)
\end{aligned}$$

which contradicts $\mathrm{E}\left[u(r^l(S(\mu,\epsilon),c))\right] = \mathrm{E}\left[u(\tilde{r}^l(S(\mu,\epsilon),c))\right]$. Analogously, one can show that $\hat{r}$ is above some finite lower bound $\underline{r}$. □

C Additional Results

Proposition 5 (Identification and implementability with limited liability)**.** *Consider a risk-neutral agent with utility, $u(r^l,c^l) = r^l - c^l$, and limited liability. Then, any generic activity $\mathring{a} = (a^l, a^{-l})$ can be implemented with a finite bonus reward, if there is no identification problem.*

Proof. There is no identification problem, so $\mu(\mathring{a}^l, \mathring{a}^{-l}) \neq \mu(\tilde{a}^l, \mathring{a}^{-l})$ for all $\mathring{a}^l \neq \tilde{a}^l$ and l. Consequently, the distribution of $S(\mu(\mathring{a}^l, \mathring{a}^{-l}),\epsilon)$ differs from that of $S(\mu(\tilde{a}^l, \mathring{a}^{-l}),\epsilon)$. In particular, there must be some set S^1 such that for all $\tilde{a}^l$: $Prob\left(S(\mu(\mathring{a}^l, \mathring{a}^{-l}), \epsilon) \in S^1\right) > Prob\left(S(\mu(\tilde{a}^l, \mathring{a}^{-l}), \epsilon) \in S^1\right)$. Define the first probability as $P(\mathring{a}^l)$ and the latter as $P(\tilde{a}^l)$. Now, pay a bonus $r^l \geq 0$ whenever $s \in S^1$ and nothing otherwise. Then, the incentive constraint of agent l becomes $r^l P(\mathring{a}^l) - c^l(\mathring{a}^l, \mathring{a}^{-l}) \geq r^l P(\tilde{a}^l) - c^l(\tilde{a}^l, \mathring{a}^{-l})$. This inequality holds if the bonus r^l is large enough: $r^l \geq \frac{c^l(\mathring{a}^l, \mathring{a}^{-l}) - c^l(\tilde{a}^l, \mathring{a}^{-l})}{P(\mathring{a}^l) - P(\tilde{a}^l)}$. The lower bound is finite for $\mathring{a}^l \neq \tilde{a}^l$. Moreover, for $\tilde{a}^l \to \mathring{a}^l$, it converges to the finite real number $D_a c^l(a)\hat{a}/D_a P(a)\hat{a}$, where $\hat{a}$ is the direction from which $\tilde{a} = (\tilde{a}^l, a^{-l})$ approaches $\mathring{a}$. Accordingly, the desired choice $\mathring{a}^l$ can be implemented with a finite bonus, r^l. □